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Theoretical Models of Earthquake Phenomena and the Physical Significance of Seismic Moment Tensor Expansions

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The principal objective of this study, among many other similar theoretical developments, has been to formulate a physically sound analytical representation of the seismic radiation field produced by spontaneous failure in an arbitrarily prestressed solid. Such formulations, usually given in terms of a Green's function integral equation, may then be expanded in a moment series which is used as a basis of interpretation of observed seismic wave fields from naturally occurring earthquakes. Consequently, a meaningful understanding of the physics involved in such an event, as can be gained from an "inversion" of observed seismic wave fields, as well as the ability to provide reliable predictions of the seismic radiation to be expected from such events, is dependent on the completeness and accuracy with which the basic integral equation (or numerically based formulation) represents the conservation laws involved in both the failure process and the energy release associated with the seismic radiation. In this regard we note that the "kinematical models" and "equivalent source models", that are frequently used, avoid consideration of the

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dynamically relevant conservation laws governing failure and energy release/absorption at the failure boundary and in the surrounding medium, so that in a fundamental sense these models are ad-hoc. Further, examination of theoretical representations of the seismic radiation from earthquakes that employ the divergence of changes in the stress within the medium as a source term in the equations of motion (such as the "stress glut model"), are shown to be without a rational physical basis and to have no meaningful relationship to the dynamics or kinematics of spontaneous failure.

In the present study we develop an approach that incorporates the (nonlinear) conservation relations on the failure surface, as well as those appropriate in the surrounding linear zone, to generate a Greens function integral equation describing both the failure growth and the (interacting) seismic radiation field. The method involves the explicit decomposition of total stress-displacement fields into dynamic and equilibrium parts, with the latter dependent on time because of the growth of a new boundary corresponding to the failure zone boundary within the prestressed medium; with this boundary growth necessitating a time dependent readjustment of the prestress state to maintain equilibrium. It is shown that the time changes in the equilibrium fields *outside* the failure zone give rise to an "equivalent force term" in the equations of motion in the linear zone corresponding to the *inertial* effect of time dependent changes in the equilibrium displacement field. Thus the "proper" equivalent force term in the equations of motion in the linear zone is this inertial term and *not* the divergence of the stress field, either outside or inside the failure zone. The Green's integral equation, arising from the equations of motion and boundary conditions (conservation relations) on the growing failure surface, show that there are three source effects that mitigate the seismic radiation field, in particular a primary term resulting from the inertial effects involving the equilibrium displacement in the medium surrounding the failure zone (a volume relaxation effect) plus secondary scattering effects from the failure surface and a final term involving energy absorption along the failure boundary, which is required for its continued growth. It is pointed out that this latter term can result in severe damping of the radiation field and is therefore of importance for accurate representations of the seismic radiation from such sources. We also show that the Green's integral equation representing the seismic radiation field can be expressed in equivalent forms, in particular in a form that is similar to that obtained earlier using an initial value formulation (eg. Archambeau and Minster, 1978) and also in the form of a combined stress pulse-dislocation source distributed along the failure boundary. One advantage of the present formulation is that it precisely prescribes the time history and spatial variability of the wave fields produced in terms of the failure zone growth rate, geometry and material properties. It is also appropriate for the general case, that is for arbitrary prestress fields and failure growth history and geometry. Therefore, a moment series expansion of the Green's integral representation, based on the directly formulated integral equation, or any of its equivalents, will produce moment coefficients that are analytically related to the fundamental parameters of the failure process. Thus the results of this study provide a basis for inference of basic source physics, as well as a physically based predictive capability for earthquake sources.

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I. Introduction

The essence of a *kinematical* moment tensor representation of earthquake seismic radiation (as opposed to a *dynamical* representation) can be phrased quite simply. In particular, the source is considered to be representable by an *equivalent body force term* in the equations of motion, so that one assumes that

$$\rho\, \partial_t^2 u_k - \partial_l \tau_{lk} = f_k$$

can represent the source effect in the equation of motion, with f_k an equivalent body force distributed over a "source volume" and of a form which will give rise to a radiation field equivalent to that of the actual source. It is also required, in effect, that the "source volume" be replaceable by an elastic region in which the usual linear Green's function representation can be used. In this context a dislocation equivalent to spontaneous failure in a stressed solid (that is failure resulting in an earthquake) can be viewed as a choice of an equivalent body force that is distributed over a planar surface within the medium. (That is, the equivalent body force has a delta function form with a magnitude that is usually taken to be proportional to an imposed displacement offset across the singular "dislocation plane".) Thus, the nonlinear effects that may actually occur within the source volume are simulated by an equivalent force **f** acting within the region or along its boundary. In this case the dynamic displacement field can be represented by:

$$u_m(\mathbf{r},t) = \frac{1}{4\pi} \int_0^t dt_0 \int_{V_0} f_k(\mathbf{r}_0,t_0)\, G_k^m(\mathbf{r},t;\mathbf{r}_0,t_0)\, d\mathbf{r}_0$$

where $G_k^m(\mathbf{r},t;\mathbf{r}_0,t_0)$ represents the Green's tensor describing elastic wave propagation in the medium and V_0 is the entire volume where f_k is non-zero (See for example Morse and Feshbach, 1953).

An example of an expansion for **u** in terms of moments of **f** may be obtained by formally expanding the Green's tensor in a Taylor's series in the source coordinates $\mathbf{r}_0$ (e.g. Stump and Johnson, 1982), so that:

$$G_k^m(\mathbf{r},t;\mathbf{r}_0,t_0) = \sum_n \left[\frac{1}{n!} G^m_{k,l_1,l_2\cdots l_n}(\mathbf{r},t;0,t_0)\right] x_{l_1}^{(0)} x_{l_2}^{(0)} \cdots x_{l_n}^{(0)}$$

where summation is implied for the repeated coordinate indices $l_1, \cdots, l_n$, and $G^m_{k,l_1 \cdots l_n}$ is evaluated at $\mathbf{r}_0 = 0$ (the origin of the source coordinates), with

$$G^m_{k,l_1 \cdots l_n} \equiv \left\{\frac{\partial}{\partial x_{l_1}^{(0)}} \cdots \frac{\partial}{\partial x_{l_n}^{(0)}}\right\} G_k^m(\mathbf{r},t;\mathbf{r}_0,t_0) \Big|_{\mathbf{r}_0 = 0}$$

Using this expansion for G_k^m in the Green's function representation for u_m then gives[+]

$$u_m(\mathbf{r},t) = \sum_{n=1}^{\infty} \frac{1}{n!} \int_0^t G^m_{k,l_1..l_n}(\mathbf{r},t;0,t_0)\, M_{kl_1 \cdots l_n}(t_0)\, dt_0$$

where

$$M_{kl_1..l_n}(t_0) = \int_{V_0} x_{l_1}^{(0)} x_{l_2}^{(0)}..x_{l_n}^{(0)} f_k(\mathbf{r}_0,t_0)\, d\mathbf{r}_0$$

is the $(n+1)$ th order moment tensor.

Generally the lowest order term $(n = 1)$, corresponding to a simple point double couple or dislocation, is used and in this case:

$$u_m(\mathbf{r},t) = \int_0^t G^m_{k,l}(\mathbf{r},t;0,t_0)\, M_{kl}(t_0)\, dt_0$$

Clearly, all of this is completely formal with no actual physical description of the source involved. Further, these representations follow from the assumption that a simple distributed equivalent force, acting in an elastic zone replacing the true physical source, can actually represent (be equivalent to) an earthquake source. That this is not necessarily so can be appreciated by reflecting on the distinct possibility that the true equivalent force may depend explicitly on the dynamic displacement field itself or on its

[+]The $n = 0$ term in the expansion for $\mathbf{u}$ vanishes by the conservation of momentum when there are no external forces or torques on the system.

space-time derivatives. In addition, in the Stump-Johnson moment expansion, it is assumed that the series expansion for G_k^m, and resulting moment series expansion for u_m, exist and do not diverge. While it may seem intuitively plausible that the latter assumptions are valid, it is also likely that expansions of this type would be slowly convergent, and certainly extremely so for a complex earthquake of large dimensions.

In a somewhat similar approach, Archambeau (1964, 1968) considered the expansion of an equivalent force representation in moments, except that, rather than formally expanding the Green's function as a series, the equivalent force was expanded as a (vector) harmonic series and the Green's function integral was evaluated as a moment series expansion. While this approach has the likelihood of producing a more rapidly convergent moment series, than would be produced by the expansion of the Green's function, it is (as was pointed out in the original development) purely formal with no direct relationship to the actual source physics.

Therefore, aside from these mathematical considerations, the essential question to be addressed is: What is the proper equivalent force term in a moment expansion, and in particular is there an equivalent force that can be related to the actual physical source. If an analytical relationship can be established, then the relationship of moment tensor observations to the physics of the source can be established.

As will be shown, the complete Green's function representation of an earthquake source results in an integral equation that can only approximately have the simple equivalent force form. But even assuming the earlier formalism to be approximately valid and a useful representation for inversion, before the formal inversion results can properly be interpreted it is still necessary to be able to relate the moment tensor components to the source physics in a rigorous fashion.

In regard to the question of the physical significance of the moment tensor, Gilbert (1970) argued on intuitive grounds that the equivalent body force equivalent could be expressed in terms of the (static) stress drop resulting from an earthquake in the form

$$f_k = -\partial_l T_{lk}$$

where T_{lk} is the stress change (throughout the earth) accompanying the seismic event. In this case, the second order moment tensor M_{lk} is just T_{lk}. Similarly, Backus and Mulcahy (1976a) attempted to provide a basis for the choice of f_k, and in so doing introduced the idea of a "stress glut". In terms of the stress glut tensor, this equivalent force is

$$f_k = -\partial_l \Gamma_{lk}$$

where Γ_{lk} denotes the "stress glut". For an earthquake source, Γ_{lk} is taken as the difference between the true physical stress, S_{lk}, and a "model stress", ζ_{lk}; with this latter stress function corresponding, within the failure zone, to the spatial continuation of the linear stress from outside the nonlinear failure zone into this zone. Hence, Γ_{lk} is the equivalent of a "stress drop" within the failure zone and, further, the second order moment tensor M_{lk} is equal to Γ_{lk} in this case. However, Γ_{lk}, by virtue of its definition, is zero in the surrounding linear zone, so that while Gilbert's equivalent is non-zero everywhere, the stress glut equivalent is non-zero only within the failure zone of an earthquake.

In this paper we will show that the moment tensor is actually fundamentally related to *time changes* in the instantaneous *equilibrium displacement* field that occurs in the medium surrounding a failure zone, and that an interpretation of the moment tensor can be obtained in terms of changes in this vector field. Furthermore, we show that the changes in the equilibrium displacements can be related, explicitly, to failure zone shape, growth rate and material rheological properties, as well as to the initial (tectonic) stress in the medium. This relationship, as might be expected, is nonlinear in *most* of the source parameters, so that inference of physical parameters from the moment tensor is a nonlinear inversion problem.

In order to develop our results with clarity in the context of current usage, it is necessary to examine the previous moment tensor derivations and interpretations in some detail. Initially, we will focus our discussion on the original derivation given by Gilbert (1970), and the critique of this formulation by Backus and Mulcahy (1976a), since many current interpretations derive from these considerations. In this regard, we will show that the moment tensor *cannot* be related to a "stress drop" in the manner described by Gilbert, and that the inclusion of gravitational effects, as described by Backus and Mulcahy in their critique of Gilbert's derivation, does *not* correct the problem. In fact, the changes in Gilbert's formulation that are advocated by Backus and Mulcahy will be shown to lead to a null result; that is the source term (and therefore the seismic radiation field) should vanish at all points and at all times in their formal representation. Gilbert's source representation, on the other hand, can be shown to *only* include the very small seismic effects due to gravity and density changes accompanying an earthquake.

In the development that follows in the next section we will first show why Gilbert's phenomenological moment tensor formulation is incorrect and why the argument involving inclusion of gravitational effects, as advanced by Backus and Mulcahy, is similarly incorrect. We will also show that Backus and Mulcahy's moment tensor representation, including the 'stress glut' representation, does not properly represent the physics of a spontaneous seismic source, such as an earthquake, nor does it properly represent an earthquake radiation field in an equivalent sense. We will then demonstrate that a basically different approach is required in order to properly describe the physics of a spontaneous seismic source. In particular, we will show how a spontaneous failure process is properly formulated *dynamically* and, when so described, provides a non-trivial moment tensor that can be related to the physics of the source.

II. Gilbert's "Phenomenological" Moment Tensor Representation

Gilbert's (1970) original development of a moment tensor representation followed the essentially intuitive approach that is still commonly evoked to model an earthquake; namely, it was assumed*

(i) That the external body force term **f** in the equations of motion:

$$\rho\, \partial_t^2 u_k - \partial_l \tau_{lk} = \rho\, f_k$$

can be used to represent the (force) effects associated with an earthquake.

(ii) That the initial state of the medium (at t = 0) is described by:

$$u_k(\mathbf{r},0) = \partial_t u_k(\mathbf{r},0) = 0$$

* Gilbert used equations of motion for a discrete system of particles and passed to the limit of a continuum for his final results. In this discussion continuum equations are used from the onset since use of the discrete system is neither necessary nor particularly useful.

(iii) That the earthquake may be adequately represented, for the purposes of predicting the normal mode response of the planet, by a step function response; so in view of the first assumption:

$$f_k(\mathbf{r},t) = F_k(\mathbf{r})\, H(t)$$

(iv) That 'the body force caused by the stress drop T_{lk} is':

$$F_k(\mathbf{r}) = -\,\partial_l T_{lk}$$

where the spatial stress drop is defined in terms of the stress *after* an earthquake, $\tau_{lk}^{(F)}$, and the stress *before* an earthquake, $\tau_{lk}^{(I)}$, as:

$$T_{lk}(\mathbf{r}) = \;_{lk}^{(I)} - \tau_{lk}^{(F)}\,;\ t > 0$$

These assumptions appear eminently plausible at first sight. However, they lead directly to a contradiction which arises from the fact that both $\tau_{lk}^{(I)}$ and $\tau_{lk}^{(F)}$ satisfy the equations of equilibrium. That is:

$$\left.\begin{aligned} \partial_l \tau_{lk}^{(I)} &= -\,\rho\, g_k^{(I)}\,;\ \mathbf{r}\ \varepsilon\ V_0\,. \\ [\![\tau_{lk}^{(I)} n_k]\!] &= 0\,;\ \mathbf{r}\ \varepsilon\ \partial V_0 \end{aligned}\right\} \qquad (1)$$

where the bracket notation applied to $\tau_{lk}^{I} n_l$ on ∂V_0, the boundary of V_0, denotes the change in the traction across the boundary. Likewise:

$$\left.\begin{aligned} \partial_l \tau_{lk}^{(F)} &= -\,\rho\, g_k^{(F)}\,;\ \mathbf{r}\ \varepsilon\ V_1 \\ [\![\tau_{lk}^{(F)} n_k]\!] &= 0\,;\ \mathbf{r}\ \varepsilon\ \partial V_1 \end{aligned}\right\} \qquad (2)$$

where $\rho g_k^{(I)}$ and $\rho g_k^{(F)}$ represent the gravitational forces acting and where the boundary conditions express continuity of tractions at (all) the medium boundaries. Here the field $\tau_{lk}^{(I)}$ is defined over the volume V_0 consisting of the entire volume of the planet, with ∂V_0 representing its boundaries, while $\tau_{lk}^{(F)}$ is defined

** In addition to the free surface of the planet, there are other boundaries within the earth where the elastic properties may change discontinuously and which, therefore, must be accounted for as internal boundaries in V_0 and V_1. However, these can be omitted from explicit mention if we observe once and for all that the tractions are continuous at all such boundaries and that at solid-solid boundaries the displacement is continuous, while on fluid-solid or fluid-fluid boundaries only the normal component of the velocity is continuous. In the present context these boundaries and the conditions on them do not change (to first order) before and after an earthquake, so the same conditions will always apply and are to be assumed throughout.

over V_1 consisting of the interior of the planet *outside* the failure zone of the earthquake, and having boundaries denoted by ∂V_1, which includes the (newly formed) failure zone boundary in addition to the boundaries ∂V_0, existing prior to the event.**

Inside the failure zone we have

$$\left.\begin{aligned} \partial_l \tau'_{lk} &= -\rho' f'_k \,;\; \mathbf{r}\,\varepsilon\, V' = V_0 \ominus V_1 \\ \tau'_{lk} n_k &= \tau^{(F)}_{lk} n_k \,;\; \mathbf{r}\,\varepsilon\,\partial V' \end{aligned}\right\} \tag{3}$$

where τ'_{lk} is any (residual) stress which may exist within the failure zone after the earthquake, while $\tau^{(F)}_{lk}$ is the final equilibrium stress in the elastic zone surrounding the failure zone and $\partial V'$ denotes the failure zone boundary. Here the symbol $\ominus$ denotes the set theoretic difference. The regions V_0, V_1 and V' and their boundaries are schematically illustrated in Figure 1.

It is important to emphasize the necessity of taking explicit account of the existence of the failure zone and to express the equilibrium equations in each zone separately, with the appropriate boundary conditions serving to connect the stress fields within the non-linear source region to those in the linear region surrounding it (i.e. the rest of the earth). This necessity arises from two facts. First. the only change in the boundaries within the earth is the formation of a failure zone boundary and, while there may be many other boundaries along which material discontinuities occur and where boundary conditions apply, this is the only boundary at which non-negligible changes occur during an earthquake. Second, and most critical for the ensuing discussion, the failure zone defines a region of non-linear behavior during its formation, within which it is not possible to apply the usual linear theory. As will be seen, part of the problem with Gilbert's representation arises from the fact that he does not account for the existence of the failure zone boundary and the associated boundary conditions and, in effect, assumes linearity everywhere throughout the planet at all times, including within the failure zone. This particular problem also arises, in a more direct fashion, in the equivalent source representation advocated by Backus and Mulcahy (1976a,b) and will be discussed later.

In order to relate the description of equilibrium fields given in (1) through (3) to Gilbert's use of equilibrium fields defined on the interior of the planet, we observe that the region over which Gilbert defines the stress drop, in (iv), is denoted as V by him and corresponds to our region V_0 (i.e. the entire planet). To properly define a stress drop T_{lk} over the entire interior of the planet it is necessary to define it as:

$$\left.\begin{aligned} T_{lk} &= \tau_{lk}^{(I)} - \tau_{lk}^{(F)}\,;\ r\,\varepsilon\,V_1 \\ T_{lk}' &= \tau_{lk}^{(I)} - \tau_{lk}'\,;\ r\,\varepsilon\,V' = V_0 \ominus V_1 \end{aligned}\right\} \tag{4-a}$$

and to include the boundary conditions applicable on the failure zone boundary $\partial V'$, in order to connect the stress drop in V' with that outside the failure zone, in V_1. This condition is simply,

$$T_{lk}n_l = T_{lk}'n_l\,;\ r\,\varepsilon\,\partial V' \tag{4-b}$$

which equates the tractions at the boundary. Thus, while the tractions are continuous, the stress drop components themselves may be *discontinuous* across $\partial V'$.

Now we can relate this stress drop definition to Gilbert's use of stress drop by simply setting the T_{lk} defined in (iv) to be equal to one or the other of the expressions in (4-a), depending on whether the coordinate point is in V_1 or V'. This modification of Gilbert's definition can be thought of as an extension of his definition, required in order that the failure zone region is explicitly (and correctly) "covered" with the correct boundary conditions included. We shall show below, however, that this is not the main difficulty with Gilbert's formal results, even though a proper definition of the failure zone equilibrium and boundary conditions is fundamental to the correct formulation of the problem.

In this regard, we note from (1) and (2) that while $\tau_{lk}^{(I)}$ and $\tau_{lk}^{(F)}$ are themselves different, because they satisfy a different set of boundary conditions (one with the failure zone present, the other without it), the divergences of these stresses can be related, since from (1) and (2):

$$\partial_l\left\{\tau_{lk}^{(I)} - \tau_{lk}^{(F)}\right\} = -\left\{\rho\, g_k^{(I)} - \rho\, g_k^{(F)}\right\};\ r\,\varepsilon\,V_1$$

by simple subtraction of the equations of equilibrium. As indicated, the relation applies over the region exterior to the failure zone (V_1). Now we see from Gilbert's assumption (iv) that the quantity in the brackets is defined as the stress drop, and so the stress drop must satisfy the relation:

$$\partial_l T_{lk} = -\left[\rho\, g_k^{(I)} - \rho\, g_k^{(F)}\right] ; \mathbf{r}\, \varepsilon\, V_1 \tag{5}$$

Here $\rho g_k^{(F)}$ represents the new gravitational force field acting after the earthquake, whereas $\rho g_k^{(I)}$ is the field before the event. The difference in these force fields is due to the redistribution of mass due to the earthquake, resulting in changes in density and associated changes in the gravitational acceleration. Gilbert does not directly concern himself with gravitational effects. (He does not neglect gravity effects as claimed by Backus and Mulcahy (1976a), he simply asserts that the body force equivalent of an earthquake is $-\partial_l T_{lk}$ and never observes that (5) is actually true.) However, if gravity effects are, in fact, neglected in comparison with effects of tectonic origin, as is shown to be justified in the Appendix 1, then:

$$\partial_l T_{lk} = 0 ; \mathbf{r}\, \varepsilon\, V_1 . \tag{6-a}$$

We also observe that the stress drop within the non-linear failure zone is also equal to the gravity field change, that is:

$$\partial_l T'_{lk} = -\left[\rho\, g_k^{(I)} - \rho' g'_k\right] ; \;\mathbf{r}\, \varepsilon\, V' ,$$

and when gravity changes are ignored relative to tectonic effects, then we also have

$$\partial_l T'_{lk} = 0 ; \;\mathbf{r}\, \varepsilon\, V' \tag{6-b}$$

within the failure zone.

Therefore we see from (6) that, to the approximation in which neglect of the gravitational forces is appropriate, the divergence of the stress drop is *zero* everywhere. In terms of Gilbert's (equivalent) 'body force caused by the earthquake', as defined in (iv), we have that this quantity is also zero. That is, we have:

$$F_k(\mathbf{r}) \equiv -\partial_l T_{lk} = 0\,;\ \mathbf{r}\ \varepsilon\ V_0\,,$$

and, as well,

$$f_k(\mathbf{r},t) = F_k(\mathbf{r})\, H(t) = 0\,;\ \mathbf{r}\ \varepsilon\ V_0\,,$$

in view of (6).

Therefore the body force term used by Gilbert to derive the moment tensor representation is, in fact, zero (or at least negligible) and the result derived from the normal mode representation using this equivalent force does not represent an earthquake displacement field. In particular, Gilbert finds that the displacement field due to a body force term of the form $f_k(\mathbf{r},t) = F_k(\mathbf{r})\,H(t)$, with $H(t)$ a step function, can be put in the form

$$\mathbf{u}(\mathbf{r},t) = \sum_n s^{(n)}(\mathbf{r}) \left\{ \int_V dV_0\, \bar{s}^{(n)}(\mathbf{r}_0) \cdot \mathbf{F}(\mathbf{r}_0) \right\} \cdot \left[\frac{1 - \cos\omega_n t \exp(-\omega_n t/2Q_n)}{\omega_n^2} \right] \tag{7}$$

where the $s^{(n)}$ are the normal mode displacement eigenfunctions for the entire "unperturbed" planet; that is the planet without the failure zone boundary present within it. Here $\bar{s}^{(n)}$ are the complex conjugates of the mode functions, ω_n are the mode angular frequencies and Q_n the associated dissipation function for each mode. (For this result, the Q_n are assumed to be much larger than unity so that dissipation is small.) This expansion is formally correct *if* effects from the failure zone boundary are neglected and *if* it is applied in the region V_1 (outside the nonlinear failure zone).

However, insertion of $-\partial_l T_{lk}$ for F_k in this representation, whether intuitively plausible or not, is *ad hoc* and quite incorrect. In fact, as we have noted, if gravity effects are negligible relative to tectonic effects (which is in fact true), then the divergence of the stress drop vanishes and so does the right side of (7). Therefore, Gilbert's application of Gauss' theorem to the volume integral term involving the divergence of the stress drop and subsequent approximations which lead to the widely used result (e.g. Geller, 1976; McCowan and Dziewonski, 1977; Dziewonski and Gilbert, 1974; Gilbert, 1973; Gilbert and Dziewonski, 1975; McCowan, 1976; Luh and Dziewonski, 1976) :

$$u_j(\mathbf{r},t) = \sum_n s_j^{(n)}(\mathbf{r}) \left\{ M_{lk} \; \bar{\varepsilon}_{lk}^{(n)} \right\} \left[\frac{1 - \cos \omega_n t \exp\left[\dfrac{-\omega_n}{2Q_n} t \right]}{\omega_n^2} \right] \tag{8}$$

is without proper foundation and the result is incorrect. (Here M_{lk} is the "moment tensor", which in this result equals the stress T_{lk}, while $\bar{\varepsilon}_{lk}^{(n)}$ is the strain tensor associated with the complex conjugate displacement eigenfunctions for the planet.) Basically, the error arises from the fact that the divergence of the stress drop is not a proper equivalent body force for an earthquake source, plausible ad hoc assertions to the contrary. The fact that this equivalent force actually vanishes when gravity effects are neglected is simply indicative of the impropriety of the assumption.

The expression in (8) is not, however, zero as would be expected. That is, since $F_k = -\partial_l T_{lk}$ vanishes (to the approximation in which gravity changes are neglected) and so causes (7) to give a null displacement field result, one might certainly expect that any result derived from (7), such as (8), should also vanish. This is not the case because, in deriving (8) from equation (7), Gilbert omits the surface integral term over the failure zone boundary so that (8) represents only part of the solution (7). The part neglected (the surface integral term) exactly cancels the right side of (8), to give the (correct) null result or, if gravity is not neglected, the displacement effects due to gravity field changes. That is, if we directly substitute the divergence of the stress drop into Gilbert's equation (7), then we get:

$$u_m(\mathbf{r},t) = \sum_n s_m^{(n)}(\mathbf{r}) \left\{ \int_V [\rho\, g_k^{(I)} - \rho\, g_k^{(F)}]\, \bar{s}_k^{(n)} dV_0 \right\} \left[\frac{1 - \cos \omega_n t \exp(-\omega_n t/2Q_n)}{\omega_n^2} \right] \tag{9}$$

since the divergence of the stress drop has the form

$$\partial_l T_{lk} = \left[\rho\, g_k^{(I)} - \rho\, g_k^{(F)} \right]$$

everywhere in the planet, including inside the failure zone. Thus, when the divergence of the static stress drop is used to represent the "equivalent force" due a spontaneous source like an earthquake, then the displacement predicted will be negligibly small or zero and not representative of the seismic radiation field produced by the event.

Backus and Mulcahy (1976a) on the other hand argue that Gilbert's results are in error precisely because of the neglect of gravity in defining a moment tensor. But this is not the reason that the result is in error, as was already indicated above. In particular, Gilbert does *not* neglect gravity since he simply *assumes* that the divergence of the stress drop is the equivalent force to be associated with an earthquake and, once this is done, gravity effects are actually implicitly included. In fact, as we have shown in equation (9), the true consequence of Gilbert's assumption is that he actually should obtain *only* that part of the displacement due to an earthquake that is associated with gravitational field changes. Further, as will be directly demonstrated, the formulation suggested by Backus and Mulcahy gives a null result when gravity is included in the way they advocate.

In particular, if the gravitational changes are included, as suggested by Backus and Mulcahy (1976a, p. 346), by defining a gravitational stress tensor (G_{ij}) such that

$$\rho\, g_j = \partial_i G_{ij} \,,$$

where

$$G_{ij} = (8\pi G)^{-1}\,(g_k g_k \delta_{ij} - 2 g_i g_j) \,,$$

then the equilibrium equations for the initial and final states, equations (1), (2) and (3), have the form:

$$\left.\begin{aligned} &\partial_l\left[\tau_{lk}^{(I)} + G_{lk}^{(I)}\right] = 0 \,;\, \mathbf{r}\ \varepsilon\ V_0 \\ &\partial_l\left[\tau_{lk}^{(F)} + G_{lk}^{(F)}\right] = 0 \,;\, \mathbf{r}\ \varepsilon\ V_1 \\ &\partial_l\left[\tau_{lk}' + G_{lk}'\right] = 0 \,;\, \mathbf{r}\ \varepsilon\ V_0 \ominus V_1 \end{aligned}\right\} . \tag{10}$$

The formal expressions given by Backus and Mulcahy (1976a, p. 347) for the equilibrium equations are, however (in our notation):+

+Backus and Mulcahy use the term "all space", which is apparently meant to be taken literally. We shall confine our discussion somewhat more to the point however, and take the liberty of replacing their "all space" by the entire interior of the earth. In our notation this is V_0.

$$\partial_l\left[\tau_{lk}^{(I)} + G_{lk}^{(I)}\right] = 0 \; ; \; \mathbf{r} \; \varepsilon \; V_0$$

$$\partial_l\left[\tau_{lk}^{(F)} + G_{lk}^{(F)}\right] = -\gamma_k^v \; ; \; \mathbf{r} \; \varepsilon \; V_0$$

where γ_k^v represents 'the equivalent volume distribution of body forces' and is asserted to be Gilbert's 'equivalent body force due to the source, corrected for gravitational effects'.

Here, since Backus and Mulcahy ignore the boundary conditions on the failure zone, as did Gilbert, and define the equilibrium equation after the failure process over the entire interior of the planet, then we must define the stress tensor for the final equilibrium state to be $\tau_{lk}^{(F)}$ when $\mathbf{r} \; \varepsilon \; V_1$ (inside the elastic zone of the planet) and to be τ_{lk}' when $\mathbf{r} \; \varepsilon \; V_0 \ominus V_1$ (inside the failure zone), as was done earlier in (4-a). Thus, if we write Backus and Mulcahy's expressions over the range of $\mathbf{r}$ for which they define it, but separately for $\mathbf{r}$ in V_1 and for $\mathbf{r}$ in V' (the failure zone), we have:

$$\partial_l\left[\tau_{lk}^{(F)} + G_{lk}^{(F)}\right] = -\gamma_k^v \; ; \; \mathbf{r} \; \varepsilon \; V_1$$

$$\partial_l\left[\tau_{lk}' + G_{lk}'\right] = -\gamma_k^v \; ; \; \mathbf{r} \; \varepsilon \; V'$$

However, comparing these directly with the required equilibrium equations given in (10) shows that:

$$\gamma_k^v \equiv 0 \; ; \; \mathbf{r} \; \varepsilon \; V_1$$

$$\gamma_k^v \equiv 0 \; ; \; \mathbf{r} \; \varepsilon \; V' .$$

That is, as must be true, equilibrium requires γ_k^v to vanish everywhere, including within the failure zone V'. Therefore it is clear that:

$$\gamma_k^v \equiv 0 \; ; \; \mathbf{r} \; \varepsilon \; V_0 = V_1 \oplus V'$$

so that this body force equivalent is, in fact, *identically zero* in both V_1 and $V' = V_0 \ominus V_1$ (that is, throughout the planet including the region within the failure zone). Thus, it contributes precisely nothing to the radiation field from an earthquake for essentially the same reason that Gilbert's stress drop equivalent force gives a null result when gravity is neglected in his formulation. Thus, *use of the divergence of changes in the equilibrium stress-gravity field as the driving force for a spontaneous source will necessarily give a null result, since the mathematical expression for the existence of equilibrium is precisely that this particular force vanish.*

As a consequence of these results, Backus and Mulcahy's definition of an extended moment tensor density as

$$M_{kl} = (\tau_{lk}^{(F)} - \tau_{lk}^{(I)}) + (G_{kl}^{(F)} - G_{kl}^{(I)})$$

and subsequent arguments about the size of the gravitational term $(G_{kl}^{(F)} - G_{kl}^{(I)})$ relative to the stress drop term $(\tau_{lk}^{(F)} - \tau_{lk}^{(I)})$ are moot, since

$$\partial_l M_{kl} = -\gamma_k^v \equiv 0$$

in any case. Thus, since the divergence of M_{kl} is the actual source term, we see that this "source term" must necessarily vanish, whatever the magnitudes of the stress drop and gravity field changes.

Backus and Mulcahy also define the boundary conditions applicable to the equilibrium states before and after the earthquake, and state them as (in our notation):

$$n_l\left[\tau_{lk}^{(I)} + G_{lk}^{(I)}\right]_-^+ = 0\,;\ \text{on}\ \partial V$$

$$n_l\left[\tau_{lk}^{(F)} + G_{lk}^{(F)}\right]_-^+ = -\gamma_k^s\,;\ \text{on}\ \partial V$$

where ∂V denotes the outer boundary of the planet (surface of the earth), since V is used by them to denote the whole interior region of the earth. Here jump notation is used, so the plus and minus signs on the brackets denote the difference in the bracketed quantity across the boundary ∂V, when approached in the limit from opposite sides. As noted earlier they ignore the failure zone boundary condition, since ∂V (which is equivalent to our boundary ∂V_0) is the surface of the earth. They introduce the surface source term γ_k^s to include the case where the failure zone intersects the surface of the earth, in which case it is *claimed* that γ_k^s will give the proper equivalent force term along this intersection on ∂V.

However, inclusion of this "surface distributed" source term, contributes nothing of substance to their formulation, and it still gives a null result. That is, consider the case when the failure zone does not happen to intersect the earth's surface. Then $\gamma_k^s \equiv 0$ by definition. Clearly then all possible equivalent source terms vanish, that is $\gamma_k^v \equiv 0$ and $\gamma_k^s \equiv 0$, and Backus and Mulcahy's formulation gives a null result. Thus, it predicts no displacement field resulting from the event whatsoever, because all source terms vanish, and this is obviously incorrect. Since this reformulation of Gilbert's representation is incorrect for all earthquakes for which the failure zone does not intersect the earth's surface, it is reasonable to conclude, without going through a detailed evaluation of the special case of a failure zone intersecting the free surface of the earth, that the reformulation is also incorrect in general. (In fact, since $\gamma_k^v \equiv 0$ everywhere in V, then it can be shown that γ_k^s must also be zero on ∂V.)

At this point we have rather thoroughly explored the so-called stress drop moment tensor representation of Gilbert and the 'extended moment tensor' representation of Backus and Mulcahy, which is intended to correct Gilbert's formulation. However, we have shown both to be misdirected and to give incorrect representations of an earthquake source radiation field. Indeed both give (essentially) null results when the required equilibrium equations are taken into account. Thus, neither of these formulations account for the physics of a spontaneous failure source, nor do they kinematically or phenomenologically represent such a source.

III. The Backus-Mulcahy Stress Glut Formulation

The main point of the work by Backus and Mulcahy (1976a,b) is not the reformulation of Gilbert's stress drop equivalent, but the formulation of a comprehensive phenomenological moment tensor representation for seismic sources in general and earthquake sources in particular. It is therefore appropriate to comment upon the basis of their representation since we intend to generate a type of moment tensor representation in a following section, but from quite a different point of view. Further, their results are very similar to Gilbert's and are similarly incorrect.

The essence of Backus and Mulcahy's approach (Backus and Mulcahy, 1976a; p. 342-346) is based on the following considerations (in their notation):

(i) The exact equations of motion and gravity for the earth are

$$\rho\, a_j = \partial_i S_{ij} - \rho\, \partial_j \psi + f_j^v \text{ in } V$$

$$n_i S_{ij} = f_j^s \text{ on } \partial V$$

$$\nabla^2 \psi = 4\pi G \rho \text{ , everywhere}$$

$$\psi \text{ and } \partial_i \psi \text{ continuous on } \partial V$$

$$\psi \rightarrow 0 \text{ , at infinity}$$

where V is the (entire) volume of the earth, ∂V its surface and f_j^v, f_j^s are externally applied body and surface forces respectively. Here ψ is the gravitational potential at any point, so $g_j = \partial_j \psi$ is the gravitational acceleration. Finally, S_{ij} are the components of the '*true physical stress*' in the earth and a_j is the particle acceleration.

(ii) A '*Mathematical model stress*' ζ_{ij} may be defined as:

$$\zeta_{ij} = E_{ijkl}\, \partial_k s_l$$

where E_{ijkl} is the ordinary elastic tensor, or as:

$$\zeta_{ij} = S_{ij}^0 - s_k \partial_k S_{ij}^0 + F_{ijkl}\, \partial_k s_l$$

when prestress S_{ij}^0 and gravity are taken into account.+ Here s_k is the (seismic) displacement in the *elastic region* of the earth. In addition, F_{ijkl} is the elastic tensor corrected for the presence of prestress, so that:++

+The term $s_k \partial_k S_{ij}^0$ is always small relative to the other terms in ζ_{ij} in seismological applications, and can always be neglected.

++Since real stresses in the earth are orders of magnitude less than the magnitudes of the elastic constants appearing in E_{ijkl}, then it is justified to take $F_{ijkl} \simeq E_{ijkl}$ in most seismological applications and, in particular, in this one.

$$F_{ijkl} = E_{ijkl} + 1/2\left[\delta_{ij}S^0_{kl} - \delta_{kl}S^0_{ij} + \delta_{il}S^0_{jk} - \delta_{jk}S^0_{il} + \delta_{jl}S^0_{ik} - \delta_{ik}S^0_{jl}\right]$$

(iii) The *model stress* can be calculated from the true displacement, s_k, by virtue of its definition and, with the introduction of the 'model stress' in the equations of motion (which can be achieved by simply adding the divergence of this 'model' stress field to both sides of the equation of motion in (i)), one obtains equations of motion expressed in terms of the *model stress* plus an extra 'source term', which is used by Backus and Mulcahy as the "equivalent driving function" or "equivalent source function" for a spontaneous release of energy corresponding to an earthquake. In particular, the equations of motion become, from (i):

$$\rho\, a_j + \partial_i\zeta_{ij} = \partial_i\zeta_{ij} + \partial_i S_{ij} - \rho\, \partial_j\psi + f^v_j \text{ , in V,}$$

when the divergence of the model stress is (formally) added to both sides of the equation of motion in (i). This equation can also be written as:

$$\rho\, a_j = \partial_i\zeta_{ij} - \rho\, \partial_j\psi + f^v_j + \gamma^v_j \text{ , in V,}$$

with,

$$\gamma^v_j = \partial_i S_{ij} - \partial_i\zeta_{ij}$$

where γ^v_j is viewed (by Backus and Mulcahy) as an equivalent *source term* arising from the introduction of the model stress ζ_{ij}. Further, the boundary condition on ∂V then becomes, in terms of the model stress:

$$n_i\zeta_{ij} = f^s_j + \gamma^s_j \text{ , on } \partial V$$

where

$$\gamma^s_j = n_i\zeta_{ij} - n_i S_{ij} \text{ , on } \partial V$$

with γ^s_j considered to be (by Backus and Mulcahy) a boundary source term, on ∂V, at the surface of the planet.

(iv) A moment tensor density M_{ij} is defined in V such that

$$\partial_i M_{ij} = -\gamma_j^v \,, \quad \text{in V},$$

$$n_i M_{ij} = \gamma_j^s \,, \quad \text{on } \partial V.$$

In addition, the "stress glut" Γ_{ij} is defined to be the difference between the model stress ζ_{ij} and the true physical stress S_{ij}, so:

$$\Gamma_{ij} = \zeta_{ij} - S_{ij} \,.$$

Backus and Mulcahy assert that *'evidently the physical source region is precisely the region where the stress glut is non-zero'* This statement therefore *fixes the choice for the model stress to be the (essentially) elastic stress outside the failure zone,* which is identical to their 'true physical stress' S_{ij} in the elastic zone.

(v) The equations of motion and gravity, given in (i) above, are linearized *throughout* the volume V (the entire planet including the failure zone volume) and a relative model stress ζ_{ij}^1 is introduced such that:

$$\zeta_{ij} = S_{ij}^0 + \zeta_{ij}^1$$

where S_{ij}^0 is the prestress in V. The equations of motion and gravity are then expressed as$^+$:

$$\rho^0 \, \partial_t^2 s_j = \partial_i \zeta_{ij}^1 - \rho^0 \, \partial_j \psi^1 - \rho^1 \, \partial_j \psi^0 + f_j^v + \gamma_j^v \,, \text{ in V},$$

$$n_i \left[\zeta_{ij}^1 + \partial_k (s_k S_{ij}^0) - (\partial_k s_i) \, S_{kj}^0 \right] = f_j^s + \gamma_j^s \,, \text{ on } \partial V,$$

$$\rho^1 = -\, \partial_k (\rho^0 \, s_k) \,, \text{ inside and outside of V},$$

$$\nabla^2 \psi^1 = 4\pi G \rho^1 \,, \text{ inside and outside of V},$$

$^+$As noted earlier in (ii), the various terms in the boundary conditions, stress tensor ζ_{ij}^1 and elastic tensor F_{ijkl}, involving the prestress S_{ij}^0 and derivatives of it, are small in the elastic zone and ordinarily are neglected. They could be large in the non-linear zone, but these linearized equations are not valid in this zone in any case.

$$\left[\psi^1\right]_-^+ = 0 \text{ and } n_i\left[\partial_k\psi^1\right]_-^+ = 4\pi G\rho^0 n_i s_i \text{ , on } \partial V,$$

$$\psi^1 \to 0 \text{ , at infinity,}$$

$$\zeta_{ij}^1 = -\, s_k\partial_k S_{ij}^0 + F_{ijkl}\partial_k s_l \text{ , in } V.$$

In addition, the initial conditions

$$s_j = \partial_t s_j = 0 \text{ , in } V \text{ at } t = 0,$$

are used.

A formal eigenfunction expansion, using the normal modes of the earth, is then used to express a solution of these linear equations as:

$$s_j(\mathbf{x},t) = \sum_{v=1}^{\infty} A_v(t)\, u_j^{(v)}(\mathbf{x})$$

with

$$A_v(t) = \omega_v^{-1}\int_0^t d\tau \sin\omega_v(t-\tau)\left\{\int_V \gamma_j^v\, \bar{u}_j^{(v)}\, dV + \int_{\partial V} \gamma_j^s\, \bar{u}_j^{(v)}\, da + \int_V f_j^v\, \bar{u}_j^{(v)}\, dV + \int_{\partial V} f_j^s\, \bar{u}_j^{(v)}\, da\right\}$$

where the integration is over the entire volume V and outer surface ∂V of the planet and $u_j^{(v)}$ are the spatial eigenfunctions for the planet, with $\bar{u}_j^{(v)}$ denoting the complex conjugate.+

The Backus and Mulcahy result for an earthquake source, where no external body forces are applied and the 'source zone' does not intersect the earth's surface ∂V, is simply

$$s_k(\mathbf{x},t) = \sum_{v=1}^{\infty} \frac{1}{\omega_v}\left[\int_0^t d\tau \sin\omega_v(t-\tau)\int_V (\partial_i\Gamma_{ij})\, \bar{u}_j^{(v)}\, dV\right] u_k^{(v)}(\mathbf{x}) \tag{11}$$

+Both Gilbert (1970) and Backus and Mulcahy (1976a) cite completeness of the modal eigenfunction set as the basis for their representations of the seismic field from an earthquake. But since they completely omit the boundary condition at the failure zone, they end up omitting integral terms in $A_v(t)$, the "excitation function", that represent scattering from the failure zone boundary, among other effects. (Such integral terms are explicitly considered in later sections of this paper and are shown to arise quite naturally when a proper Green's function integral representation is used to describe the seismic radiation effects associated with a spontaneous failure process - that is an earthquake.) Further, and most important, these auin terms of the 'stress-glut'. Here the spatial integration is *only* over the 'source region', that is V' the failure zone, and the 'model stress' is the stress in the linear region. (If the 'source region' intersects the free surface ∂V, then their result would also include the surface integral over ∂V involving $\gamma_j^s = n_i\Gamma_{ij}$.)

The logical development described in (i) through (v) is not, however, a proper one and the 'stress glut' and moment tensor representations involving volume integration over the failure zone are, consequently, incorrect.

The precise nature of the problem with their approach can be seen most clearly by considering the failure zone, and its associated boundary, explicitly in the formulation. We employ the notation used earlier and indicated in Figure 1. Appropriate equations of motion in the separate regions V' (the nonlinear failure zone) and V_1 (the linear zone surrounding the failure zone) are

$$\left.\begin{aligned} \frac{d}{dt}\left[\rho' v_i'\right] - \partial_j S_{ij}' &= \rho' g_i' \;;\; \mathbf{r} \,\varepsilon\, V' \\ \rho_1 \partial_t v_i^{(1)} - \partial_j S_{ij}^{(1)} &= \rho_1 g_i^{(1)} \;;\; \mathbf{r} \,\varepsilon\, V_1 \end{aligned}\right\} \tag{12}$$

with the tractions, $S_{ij}'n_j$ and $S_{ij}^{(1)}n_j$, and velocity fields, v' and $v^{(1)}$, in V' and V_1 being related by boundary equations expressing conservation of momentum, mass and energy across $\partial V'$, the failure zone boundary. On the other hand we see from (i) above that Backus and Mulcahy write the equations of motion throughout the planet in the form

$$\rho\, a_i - \partial_j S_{ij} = \rho g_i \tag{13}$$

with a_i the acceleration. (Applied external forces are not relevant and are omitted throughout the remaining discussion.) This equation can be formally related to those in V' and V_1 by simply letting the equation take the form of one or the other of the equations in (12), depending on whether $\mathbf{r}$ is in V' or V_1. Since Backus and Mulcahy use the stress in the linear zone, $S_{ij}^{(1)}$, as their model stress (which, in fact, is the *only* choice) then, as in (iii) above, this gives for their equation (13):

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thors consequently ignore the essential nonlinear properties of the failure zone and integrate through this region, thereby summing (physical or "true") stress changes and gravity field changes from within this nonlinear zone as if they were linear effects which superpose linearly and are propagated linearly within this zone.

$$\rho\, a_i - \partial_j S_{ij}^{(1)} = \rho\, g_i + \gamma_i^v \tag{14}$$

with

$$\gamma_i^v = \partial_j (S_{ij} - S_{ij}^{(1)})$$

Now in terms of the equations given in (12) for the linear and non-linear regions, this referencing to $S_{ij}^{(1)}$ simply gives:

$$\left.\begin{aligned} &\frac{d}{dt}\left[\rho' v_i'\right] - \partial_j S_{ij}^{(1)} = \rho' g_i' + \partial_j\left[S_{ij}' - S_{ij}^{(1)}\right] \;;\; \mathbf{r}\, \varepsilon\, V' \\ &\rho_1 \partial_t v_i^{(1)} - \partial_j S_{ij}^{(1)} = \rho_1 g_i^{(1)} \;;\; \mathbf{r}\, \varepsilon\, V_1 \end{aligned}\right\} \tag{15}$$

The set of equations in (15) is the explicit equivalent of the equation (14), which is that used by Backus and Mulcahy.

Backus and Mulcahy then define the stress difference $S_{ij}^{(1)} - S_{ij}'$, appearing in (15), to be the 'stress glut' Γ_{ij}. Next, as described in (v) above, they *linearize* the equations of motion everywhere in V, the interior of the planet, so that in (14) they in effect replace ρa_i by $\rho\, \partial_t^2 s_i$. However, as we see from (15), this requires the same approximation to be carried out in the nonlinear region V'. That is, their linearization of equation (14) throughout the planet means that they linearize the first of the equations in (15) within the non-linear zone. But obviously the acceleration term $\frac{d}{dt}\left[\rho' v_i'\right] = \frac{d}{dt}\left[\rho' \frac{ds_i'}{dt}\right]$ *cannot* ordinarily be approximated by $\rho' \partial_t^2 s_i'$ in such a region, since transport terms, which may be large compared to $\rho' \partial_t^2 s_i'$, are neglected [+]. However, even if such an approximation is made for this inertial term the result is:

$$\rho' \partial_t^2 s_i' - \partial_j S_{ij}^{(1)} = \rho' g_i' + \partial_j\left[S_{ij}' - S_{ij}^{(1)}\right] \;;\; \mathbf{r}\, \varepsilon\, V' . \tag{16}$$

But Backus and Mulcahy use the same displacement function s_i everywhere throughout the planet. That is, they effectively replace the inertial term $\rho' \partial_t^2 s_i'$ in (16) by $\rho_1 \partial_t^2 s_i^{(1)}$, which corresponds to the analytic continuation of the linear (elastic) acceleration field into the nonlinear failure zone. Therefore, Backus

[+]Transport terms are those arising from the last term in the identity: $d/dt\,(\rho \mathbf{v}) = \partial_t\,(\rho \mathbf{v}) + \mathbf{v}\cdot\nabla(\rho \mathbf{v})$. Thus when the total derivative in the identity is approximated by the partial derivative, then this requires neglect of terms like $\mathbf{v}\cdot\nabla(\rho \mathbf{v})$.

and Mulcahy use a fully linearized equation in the failure zone, in particular:

$$\rho_1 \partial_t^2 s_i^{(1)} - \partial_j S_{ij}^{(1)} = \rho_1 g_i^{(1)} + \partial_j (S_{ij}' - S_{ij}^{(1)}) \;\; ; \;\; \mathbf{r} \in V' \tag{17}$$

Thus, these authors not only implicitly linearize the inertial term in equation (15), but they also replace the nonlinear displacement s_i' with the analytic continuation of the linear displacement $s_i^{(1)}$ from outside the failure zone *into* this zone. Further, they similarly replace ρ' and g_i' by ρ_1 and $g_i^{(1)}$, again by implicit continuation. Viewed in terms of the validity of such a procedure, even as the crudest of approximations, it seems clear that it has no physical or logical basis. Certainly if fully nonlinear problems could be solved by introducing a source term which incorporated a part of the nonlinear equation (in this case the nonlinear stress term), with the remainder of the equation linearized so that a linear Green's function solution could be obtained, then we would have at our disposal a method that could solve any nonlinear problem. There is no question that this is not the case, neither in general nor in the present problem.

In fact, if the procedure employed by Backus and Mulcahy is carried to its correct logical conclusion, then the differences between the nonlinear inertial and gravitational terms in V' and the analytic continuation of the comparable linear terms from outside the failure zone into V', should be included in the source factor. That is, by the same logic used to introduce the stress difference term (the "stress glut") in the equations of motion in V', one can form from (15) (by adding and subtracting the analytically continued inertial and gravity terms $\rho_1 \partial_t^2 s_i^{(1)}$ and $\rho_1 g_i^{(1)}$) the equation:

$$\rho_1 \partial_t^2 s_i^{(1)} + \left[\frac{d}{dt} \left[\rho' v_i' \right] - \rho_1 \partial_t^2 s_i^{(1)} \right] - \partial_j S_{ij}^{(1)} = \rho_1 g_i^{(1)} + \left[\rho' g_i' - \rho_1 g_i^{(1)} \right] + \partial_j \left[S_{ij}' - S_{ij}^{(1)} \right] \; ; \mathbf{r} \in V'$$

This is perfectly rigorous and no approximations are involved, as opposed to the Backus-Mulcahy result. Now the extra factors appearing, in addition to the "stress glut" term, are the two difference terms in brackets. These can, following the reasoning of Backus and Mulcahy, be considered as (unknown) "source" terms and, to emphasize this interpretation, can be written in combination with the stress glut term as a "source" of the linear elastic field $s_i^{(1)}$. That is, the previous equation can be rearranged as:

$$\rho_1\,\partial_t^2 s_i^{(1)} - \partial_j\,S_{ij}^{(1)} = \rho_1\,g_i^{(1)} + \left\{\rho_1\,\partial_t^2 s_i^{(1)} - \frac{d}{dt}\,(\rho_1\,v_i') + (\rho' g_i' - \rho_1\,g_i^{(1)}) + \partial_j\,(S_{ij}' - S_{ij}^{(1)})\right\}\ ;\ \mathbf{r}\varepsilon V'$$

Further, one can combine this result with the second equation in (15) to obtain a result applicable in "all space", that is for $\mathbf{r}\ \varepsilon\ V'\oplus V_1$. In particular, one has the formal result:

$$\rho_1\,\partial_t^2\,s_i^{(1)} - \partial_j\,S_{ij}^{(1)} = \rho_1 g_i^{(1)} + \gamma_i \quad ;\mathbf{r}\ \varepsilon\ V'\oplus V_1$$

where

$$\gamma_i = \left\{\left[\rho_1\partial_t^2\,s_i^{(1)} - \frac{d}{dt}(\rho' v_i')\right] + \left[\rho' g_i' - \rho_1 g_i^{(1)}\right] + \partial_j(S_{ij}' - S_{ij}^{(1)})\right\} \quad ;\mathbf{r}\ \varepsilon\ V'$$

$$\gamma_i = 0 \quad ;\mathbf{r}\ \varepsilon\ V_1$$

This is essentially the same kind of result as that obtained by Backus and Mulcahy, but includes the two extra factors in the source term γ_i that were neglected by them.

At this point one might be tempted to conclude, since this is rigorous with nothing neglected, that the result constitutes a very similar but somewhat more "precise" representation of the "equivalent source" term. However, while it is true that the equation is, in fact, the correct consequence of the procedure used by Backus and Mulcahy, it by no means provides a representation of the equivalent source due to spontaneous failure (an earthquake) or any other kind of source. This is easy to see, since it is only necessary to note from the first of the equations in (15) that:

$$\frac{d}{dt}\,(\rho' v_i') - \partial_j S_{ij}' = \rho' g_i' \quad ;\mathbf{r}\ \varepsilon\ V'$$

which only expresses conservation of momentum in V'. But if this equality must be satisfied, which of course it must, then several of the terms in the expression for γ_i must cancel. In particular, using this equation we have that γ_i must always (identically) reduce to:

$$\gamma_i = \left\{\rho_1\partial_t^2 s_i^{(1)} - \partial_j S_{ij}^{(1)} - \rho_1 g_i^{(1)}\right\} \quad ;\mathbf{r}\ \varepsilon\ V'$$

$$\gamma_i = 0 \quad ;\mathbf{r}\ \varepsilon\ V_1$$

If this is now re-inserted into the "equation of motion in all space", we have:

$$\rho_1\partial_t^2 s_i^{(1)} - \partial_j S_{ij}^{(1)} = \rho_1 g_i^{(1)} + \left\{\rho_1\partial_t^2 s_i^{(1)} - \partial_j S_{ij}^{(1)} - \rho_1 g_i^{(1)}\right\} \quad ; \mathbf{r}\ \varepsilon\ V'$$

$$\rho_1\partial_t^2 s_i^{(1)} - \partial_j S_{ij}^{(1)} = \rho_1 g_i^{(1)} \quad ; \mathbf{r}\ \varepsilon\ V_1$$

While these equations are exact, the first is nothing but a trivial identity and the second is the regular equation of motion in the elastic medium outside the source region. Thus, the consequence of the approach used by Backus and Mulcahy, when carried out properly, is a triviality. However, since they actually neglect terms of the same order as the "stress glut" term they retain, they do not recognize the circular nature of their procedure. In any case, it is clearly safe to conclude that the formal mathematical basis for their representation is erroneous.

IV. Consequences of the Stress Glut Phenomenological Representation

Aside from the previous observations concerning the foundations of the stress glut representation, there are several consequences of this formulation that indicate its inappropriate nature.

As a first example, consider the limiting form of the stress glut function as time tends to infinity (the static limit). In this case, with $\gamma_i^v \equiv \partial_j \Gamma_{ij}$:

$$\lim_{t\to\infty} \gamma_i^v = \lim_{t\to\infty} \left[\partial_j S_{ij}'\right] - \lim_{t\to\infty} \left[\partial_j S_{ij}^{(1)}\right] = \begin{cases} \rho' g_i' - \rho_1 g_i^{(F)} ; \mathbf{r}\ \varepsilon\ V' \\ 0 ; \mathbf{r}\ \varepsilon\ V_1 \end{cases}$$

where ρ', etc., represent final static values of density, etc. Here we have simply used the fact that the static stresses must satisfy equilibrium equations. Therefore we observe that, like Gilbert's representation, this source term *only* represents gravity field changes in the long time, or low frequency, limit. However, while Gilbert's result represented earthquake induced gravity changes in the entire planet, the result given by Backus and Mulcahy represents, in the low frequency limit, only the changes *within* the failure zone. But, as previously observed, the total of all gravity change effects are at least several orders smaller than tectonic effects and the effect of those within the failure zone alone would be considerably less than those to be obtained from Gilbert's result.

More explicitly, if we evaluate the displacement field due to the "stress glut", as given by Backus and Mulcahy in equation (11), taking the stress functions involved in the stress glut tensor to be at least approximately separable in space and time, then we get:

$$s_k^{(1)}(\mathbf{x},t) = \sum_{\nu=1}^{\infty} \frac{u_k^\nu}{\omega_\nu} \left[\int_V (\rho' g'_j - \rho_1 g_j^{(1)})\, u_j^\nu \, dV \right] \int_0^t \sin \omega_\nu (t-\tau)\, f(\tau)\, d\tau$$

where f(t) denotes a time function characterizing the temporal variation of both the stresses S'_{ij} and S_{ij} in the definition of the stress glut. Thus, no matter what the form of the time variation of the stress glut factor, the magnitude of the predicted displacement field is determined by the inner product of the gravity field changes with the elastic eigenfunctions for the earth taken over the failure zone. As with Gilbert's result, with which it differs only in that the volume integration is over the failure zone rather than the entire volume of the earth, the predicted displacement field from this representation will be many orders of magnitude less than that actually associated with earthquakes.

The use of the stress glut formulation in the context of inclusion theory (*e.g.* Eshelby, 1957), affords another example of a clearly incorrect prediction for the elastic wave radiation. Here the problem, as considered by Backus and Mulcahy, involves the creation of a volume inclusion within a stressed medium where the material within the inclusion is viewed as transforming to a new physical state, such as from a solid to a fluid +. A transformation of this sort must result in the radiation of elastic waves, since the stress outside the inclusion must dynamically adjust to the presence of the new material. In treating this problem Backus and Mulcahy use the expression

$$\Gamma_{ij} = E_{ijkl} e_{kl}^F$$

for the stress glut, where E_{ijkl} is taken to be the elastic tensor throughout the inclusion *after* transformation and e_{kl}^F is the so-called "stress-free strain", which is precisely the strain that would occur in the inclusion if it were allowed to undergo the transformation unconstrained by the surrounding matrix (*i.e.*, when the

+ The time variation for this transformation is unspecified in the problem.

inclusion boundary is a free surface).

However, for the transformations described by the Eshelby inclusion theory, E_{ijkl} must be a constant (Eshelby, 1957). Further e^F_{kl} is by definition an equilibrium field. Consequently, the equivalent volume force vector associated with this stress glut vanishes. That is:

$$\gamma_j^v \equiv \partial_j \Gamma_{ij} = \frac{\partial}{\partial x_j}\left[E_{ijkl} e^F_{kl}\right] = 0$$

since the elastic tensor is constant in the inclusion and e^F_{kl} is an equilibrium strain field by construction. Therefore this stress glut representation gives a null result for this problem and this obviously cannot be correct.

It is also easy to see that a stress glut formulation does not describe the physical manifestations of phase transitions. In particular, consider the transition to involve sudden melting to form an inclusion within a solid matrix that is initially under a pure shear prestress. Suppose the melting is such that an ideal fluid is produced, so that the shear modulus is zero. Then E_{ijkl} reduces to a form such that

$$\Gamma_{ij} = \lambda \delta_{ij} e^F_{kk}$$

where λ is the Lamé constant for the fluid and e^F_{kk} is the volume dilation of the fluid when the boundary stresses due to the surrounding matrix are removed. From the form of the stress glut it is evident that it corresponds to a pure pressure, with no shear stress components. Thus, we can conclude that this source can never produce a correct result, since not only does the divergence of the quantity Γ_{ij} vanish but the only radiation field that could possibly be produced by such a source term would be a pure compressional wave field since only e^F_{kk} is present and we know that shear waves must be produced because the initial field in the surrounding matrix was pure shear and it must change drastically after the transformation. (Such a change can only be accomplished by the radiation of shear waves, which result in a relaxation of the shear stresses in the vicinity of the fluid inclusion.) Therefore, aside from the earlier fundamental considerations, even a rather cursory examination of these special cases shows that the stress glut source representation fails to describe the basic properties of the elastic wave radiation to be expected.

V. Dynamical Representation of Spontaneous Failure in a Stressed Medium: The "Time Dependent Equilibrium" Method

In order to display the proper "equivalent force" term for a spontaneous seismic source like an earthquake, which derives energy for its formation from the medium itself and simultaneously releases stored energy from the medium, as evidenced by the seismic waves radiated from the event, it is necessary to carefully consider the nature of equilibrium in the medium during formation of a failure zone. In doing so we observe from the onset that the conditions for equilibrium are continually changing with time as the failure zone forms and expands. This follows from the force balance requirements themselves and, in particular, from the fact that equilibrium depends upon continuity conditions at all the boundaries of the medium. Therefore since failure produces a boundary within the medium that changes dimensions with time, then the equilibrium stress must also change parametrically with time because of the changing boundary conditions applying to the internal stress fields.

More quantitatively these statements regarding equilibrium changes are expressed by the relations [+]:

$$\left.\begin{aligned} \partial_l \tau_{lk}^{(e)} &= -\rho^{(e)} g_k^{(e)} \; ; \; \mathbf{r}_s \in V_1(t_s) \\ [\![\tau_{lk}^{(e)} n_l]\!] &= 0 \; ; \; \mathbf{r}_s \in \partial V_1(t_s) \end{aligned}\right\} \qquad (18)$$

where we use a superscript (e) to explicitly label the variables as equilibrium fields. Further, we choose to explicitly label the coordinates as $\mathbf{r}_s$ and t_s, denoting "source spatial coordinates" and a "source time coordinate", respectively. These equations are entirely analogous to those of equation (2), except that we have taken explicit note of the fact that the volume external to the failure zone, V_1 as shown in Figure (1), and the surface of this volume, ∂V_1, are both parametrically dependent on the time variable t_s. Thus the equations in (18) express the conditions for equilibrium in the elastic region and since $V_1(t_s)$ and $\partial V_1(t_s)$ depend on time, then the solution of these equations will yield stress, density and gravity fields that are

[+]As before, the double bracket notation appearing in (18) is used to denote the jump in the enclosed quantity across the boundary.

functions of both $\mathbf{r}_s$ and t_s. This dependence will occasionally be explicitly displayed, for example by writing the stress $\tau_{lk}^{(e)}$ as $\tau_{lk}^{(e)}(\mathbf{r}_s, t_s)$ and similarly for the other field variables. Likewise, the equations specifying equilibrium inside the failure zone are

$$\left.\begin{aligned} \partial_l^{(1)}\tau_{lk}^{(e)} &= -\rho^{(e)} g_k^{(e)} \,;\, \mathbf{r}_s \in V'(t_s) \\ [\![{}^{(1)}\tau_{lk}^{(e)} n_l]\!] &= 0 \,;\, \mathbf{r}_s \in \partial V'(t_s) \end{aligned}\right\} \tag{19}$$

where V' is the failure zone volume and $\partial V'$ its surface, as shown Figure (1). This equation expresses the same equilibrium relationships as were expressed by equation (3), but here again we have been more explicit in indicating the parametric dependence of the fields on time through the linkage with the time dependent volume and surface boundary of the failure zone.

Of course it is not usual to speak of an equilibrium field as depending on time and it appears at first sight to be a contradiction in terms. However there is really no contradiction, since the dependence is parametric; that is these fields are the equilibrium fields that *would exist* if the failure zone boundary were of the size and shape specified at a particular time t_s. Thus, we are merely asking what the equilibrium fields would be if the failure zone were "frozen", for all time, at a size and shape appropriate to a particular time t_s. Since the size and shape of the failure zone is a function of the time variable t_s and since the equilibrium fields (stress or displacement) are a function of coordinates of the surface at time t_s, then the equilibrium fields will be implicit functions of t_s. Further, we know that the spatial dependence of an equilibrium displacement field is always a bi-harmonic function with coefficients of the bi-harmonic series depending on the surface coordinates of the failure zone inclusion. Thus we also know, a-priori, that the parametric dependence on t_s will appear in the coefficients of a bi-harmonic series expansion for the equilibrium displacement.

In the dynamic description of the phenomena associated with spontaneous failure in a stressed medium the time dependence of the equilibrium field is a hidden (or internal) variable. That is, the equations of motion in the linear region outside the failure zone are

$$\partial_{t_s}(\rho\, \partial_{t_s} u_k) - \partial_l \tau_{lk} = \rho\, g_k \; ; \; k = 1, 2, 3 \tag{20}$$

when written with the source coordinates as independent variables. However, the various dependent variables, such as the displacement field $u_k(\mathbf{r}_s, t_s)$, are total fields; that is they include both the equilibrium field and a purely dynamic field as components. In particular, they are of the form

$$u_k = u_k^{(e)} + u_k^{(d)} \tag{21}$$

where the "dynamic field", $u_k^{(d)}$, can be defined simply as the difference between the field u_k appearing in (20) and the field $u_k^{(e)}$ defined by (18). Clearly, (20) does not display the equilibrium field $u_k^{(e)}$ explicitly and so it, along with the other equilibrium fields connected with this equation, can be termed hidden internal field variables.

Ordinarily the equilibrium fields are of no consequence in the solution of (20), since in most elastodynamic problems the equilibrium displacement is not a function of time. Thus, in the case when the equilibrium is constant, substitution of the complete form for u_k into (20) produces $\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(d)})$ for the inertial term and substitution of the comparable expressions for τ_{lk} and g_k, involving the sum of their equilibrium and dynamic components, results in cancellation of the resulting equilibrium terms by virtue of the equations of equilibrium, as given by equations of the form of (18). Therefore in the case of time *independent* equilibrium, equation (20) is an equation for the dynamic fields and states that:

$$\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(d)}) - \partial_l \tau_{lk}^{(d)} = \rho\, g_k^{(d)} \; ; \; k = 1, 2, 3$$

However, when the equilibrium fields are time dependent, as is the situation for a spontaneous failure process, then equation (20) gives a different result. In this case we have

$$\left.\begin{aligned} u_k &= u_k^{(e)}(\mathbf{r}_s, t_s) + u_k^{(d)}(\mathbf{r}_s, t_s) \\ \tau_{lk} &= \tau_{lk}^{(e)}(\mathbf{r}_s, t_s) + \tau_{lk}^{(d)}(\mathbf{r}_s, t_s) \\ g_k &= g_k^{(e)}(\mathbf{r}_s, t_s) + g_k^{(d)}(\mathbf{r}_s, t_s) \end{aligned}\right\} \tag{22}$$

where the equilibrium fields all depend on t_s by virtue of equations (18) and (19). Now, use of these forms in (20) gives:

$$\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(d)}) + \partial_{t_s}(\rho\, \partial_{t_s} u_k^{(e)}) - \partial_l \tau_{lk}^{(d)} - \partial_l \tau_{lk}^{(e)} = \rho\, g_k^{(e)} + \rho\, g_k^{(d)}$$

Here the inertial term involving $u_k^{(e)}$ does not necessarily vanish, since there will be variations in $u_k^{(e)}$ because of the time dependent boundary changes. However, as previously noted in equation (18):

$$\partial_l \tau_{lk}^{(e)} = -\rho g_k^{(e)}$$

and this relation holds in all situations; that is whether $\tau_{lk}^{(e)}$ and $g_k^{(e)}$ are parametrically time dependent or not. Therefore we have,

$$\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(d)}) - \partial_l \tau_{lk}^{(d)} = -\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(e)}) + \rho\, g_k^{(d)}\ ;\ \mathbf{r}_s \in V_1(t_s)$$

The equilibrium field in the linear zone outside the failure volume can, however, be expressed as the sum of the changes in the equilibrium state due to the introduction of the failure zone, which can be denoted by $\mathbf{u}^{(e)'}(\mathbf{r}_s\ ,\ t_s)$, plus the (time independent) field $\mathbf{u}^{(I)}(\mathbf{r}_s)$ describing the initial equilibrium state. Thus we can always express $\mathbf{u}^{(e)}$ as

$$\mathbf{u}^{(e)}(\mathbf{r}_s\, , t_s) = \mathbf{u}^{(e)'}(\mathbf{r}_s\ ,\ t_s) + \mathbf{u}^{(I)}(\mathbf{r}_s)\ ;\ \mathbf{r}_s\, \varepsilon\, V_1$$

as a consequence of linearity. Since time derivatives of $\mathbf{u}^{(I)}$ vanish, then the equation of motion becomes

$$\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(d)}) - \partial_l\, \tau_{lk}^{(d)} = -\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(e)'}) + \rho g_k^{(d)}\ ;\ \mathbf{r}_s\, \varepsilon\, V_1(t_s) \tag{23}$$

Thus, we have that the term $-\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(e)'})$ is the source term (or "equivalent body force term") for the dynamic field generated by a localized failure process within a prestressed solid. Clearly this source term is distributed throughout the region V_1, since it involves time rate changes of the equilibrium displacement defined throughout this region. Thus, the equivalent body force term for an earthquake is not the divergence of a stress drop, but is an "inertial force" produced by changes in the elastic *equilibrium* displacement field that occur because of the creation and growth of the failure zone in the prestressed solid. More specifically, if the failure zone growth occupies a time interval $(0, \tau_0)$, then neglecting the small effects of density changes associated with the relaxation of stresses:

$$-\partial_{t_s}(\rho\, \partial_{t_s} u_k^{(e)'}) = -\rho\, \partial_{t_s}^{\,2} u_k^{(e)'} = \begin{cases} O\ ;\ t_s < 0 \\ F(\mathbf{r}_s, t_s) \neq 0\ ;\ 0 \le t_s \le \tau_0 \\ 0\ ;\ t_s > \tau_0 \end{cases}$$

where the "force" function $F(\mathbf{r}_s, t_s)$ is a direct function of the failure volume time history of growth. This function must also depend on the rheology of the material within the failure zone as well as the elastic properties of the medium surrounding it, by virtue of the boundary conditions at the failure surface.

The analytical form of this source term is prescribed by the boundary value problem of (18). Therefore, since the fields prescribed by (18) are equilibrium fields, it follows that the displacement field $\mathbf{u}^{(e)}$ is a biharmonic function of the spatial coordinates (e.g. Landau and Lifshitz, 1951), as noted earlier. Further, the field $\mathbf{u}^{(I)}$ and $\mathbf{u}^{(e)'}$ are similarly biharmonic. Thus, since the coefficients of the biharmonic series are fixed by the shape of the failure zone inclusion as well as by the value of the initial stress field and the rheology of the material (both within and outside the failure zone), then the parametric time dependence of $\mathbf{u}^{(e)'}$ will be contained within these series coefficients. Consequently, even without knowledge of the prestress field, the rate of failure expansion, the material properties and the shape of the failure zone, we know a good deal about the analytical form of this equivalent force term.

Before making use of this formulation for the dynamics of failure phenomena (and our knowledge of the biharmonic form of $\mathbf{u}^{(e)'}$) it is useful to point out that this formulation is a particular example of a more general class of problems involving time dependent internal equilibrium changes. In this regard, it has been shown by Dilts (1985) that the decomposition of stress and displacement fields into dynamic and equilibrium components, so as to display the internal equilibrium field variable, allows the permanent deformations due to microcracking or lattice dislocation creation or movement (which results in macroscopic plastic behavior) to be treated directly in terms of equilibrium field changes in the equations of motion for the material. As with the present problem, time dependent equilibrium changes appear as dynamical terms in the equations of motion. However, as shown by Dilts, since the particular equilibrium field changes associated with microscopic disordering within the solid changes are generally dependent on the dynamical displacement $\mathbf{u}^{(d)}$ and its derivatives, then the result is the occurrence of non-linear terms in the equations of motion.

On the other hand, we observe that the result expressed by (23) is a linear equation. This is because we can partition the spatial region around a single failure zone into zones of linear and nonlinear material behavior, where the changes in equilibrium within the linear (elastic) zone only occur because of the requirement that boundary conditions must be satisfied on the time varying failure boundary. Therefore, the equilibrium field, $\mathbf{u}^{(e)'}$, at a point in V_1 does not depend on the dynamic displacement or its derivatives *at that point*, but only on geometric and material characteristics at the failure boundary. As a consequence the equilibrium term in (23) is linear. However, the behavior of the material within the failure zone would be described by internal equilibrium field changes that depend on the local displacement field and its deratives due to the intense microscopic disordering produced by any failure process. Consequently the dynamics of the material motion in the failure zone would be described by an equation of motion with dynamic terms again involving changes in internal equilibrium, but now with these terms depending on the *local* dynamic displacement and its derivatives, so that these terms would be nonlinear in the dynamic field variable.

We note further that both linear and non-linear changes in equilibrium can result in either absorption of energy from the dynamic field or release of energy to it. That is, "dynamic equilibrium" terms in the equations of motion can correspond to either "sources" or "sinks" of energy for the dynamic field. Further, this behavior may not only be spatially dependent but also time dependent, so that energy may be absorbed or stored at a point and later partially or totally released at that same point. Clearly, because of the irreversible nature of the nonlinear changes within a failure zone, we expect absorption of energy in this region while in the linear zone we expect a net reduction of stored energy in the equilibrium field, with the stored energy changes being a consequence of relaxation of the equilibrium tectonic prestress field around the failure zone. Clearly then, the contribution of the equilibrium field changes as "sources" for the dynamic field can be very complex and can involve temporary increases in stored energy, followed by decreases, since the strain changes are reversible in the linear zone and the growing failure zone can require increased strain levels locally followed by decreases as the position of the point relative to the failure boundary changes. Such behavior will be dictated, on a spatial and temporal basis, by the analytical form of $\mathbf{u}^{(e)'}$ in equation (23), with the time dependent biharmonic coefficients in the expansion for $\mathbf{u}^{(e)'}$ reflecting the changing position of any point in the medium relative to the failure boundary as it grows.

In addition to the rather complex linear behavior to be expected from the volume source term in equation (23), it is also to be expected that dynamical complications will arise at the boundary of the failure zone. Here we expect, on physical grounds, that energy will be transferred from the linear zone to the nonlinear zone where it will be absorbed by one irreversible process or another. In particular, it can be expected that the energy required to cause failure will be, at least in part, extracted from the dynamical field at the boundary. Such phenomena must therefore be reflected in the boundary conditions connecting the linear and non-linear zones at the failure boundary. Thus such boundary conditions, along with (23), serve to define the complete dynamical boundary value problem for the displacement field in the linear region V_1.

The boundary conditions that are appropriate are those that express conservation of mass, momentum and energy across such a "singular" surface, where the possibility of irreversible energy absorption along a moving boundary is explicitly addressed. This physical situation has been considered by Archambeau and Minster (1978) and they have shown that the appropriate boundary conditions on the failure surface, $\partial V'$, are:

$$\left.\begin{aligned} &[\![\rho v_i^* n_i]\!] = 0 \\ &[\![(\rho v_k v_i^* - \tau_{ki}) n_i]\!] = 0 \\ &[\![(\rho E\, v_i^* - v_k \tau_{ki} + q_i) n_i]\!] = 0 \end{aligned}\right\} \tag{24}$$

where the double bracket denotes the change in the quantity enclosed across the failure surface. Thus, for example:

$$[\![v_i]\!] \equiv v_i(\partial V_I') - v_i(\partial V_O')$$

where $\partial V_I'$ and $\partial V_O'$ denote the surface $\partial V'$ approached from inside and outside the failure zone.

In these relations v denotes the particle velocity $\left[\frac{du}{dt}\right]$, **n** is the normal to the failure zone boundary and **q** the heat flux vector. Also

$$\mathbf{v}^* \equiv \mathbf{v} - \mathbf{U}$$

where **U** is the velocity of the failure surface boundary and E is the total energy density for the material, defined by

$$E \equiv \Gamma + v_k v_k/2 + \phi$$

with Γ the internal energy and ϕ the gravitation energy. The quadratic $v_k v_k/2$ corresponds to the kinetic energy contribution.

All the field variables appearing in (24) are, of course, functions of the coordinates and time, including the surface normal **n** and **U**, the rate of change of the failure boundary with time. The first of the equations in (24) expresses conservation of mass, the second conservation of momentum and the last conservation of energy. All the field variables include equilibrium as well as dynamic components. Thus, for example, the particle velocities have the explicit form:

$$\mathbf{v} = \mathbf{v}^{(d)} + \mathbf{v}^{(e)}$$

As was shown by Archambeau and Minster, the boundary conditions can be reduced to simpler form for the physical situation prevailing during rapid failure. In particular, when the density changes (and the gravitational change ϕ) upon failure are small and the rupture rate is high (that is equal to a significant fraction of the local shear velocity and much larger than the particle velocity v) then, when the heat flux changes across the failure surface are neglected relative to the larger mechanical flux term $[\![v_k \tau_{ki}]\!]$, it follows that the equations in (24) reduce to:

$$\rho U_R [\![v_k]\!] = - [\![\tau_k]\!] \tag{25}$$

where $\tau_k = \tau_{ki} n_i$ are the components of the surface tractions on the failure surface. Equation (25) combines the conservation of mass and momentum equations from (24). Further, under rapid failure conditions the energy equation in (24) reduces to:

$$[\![\tau_k \tau_k]\!] = 2\rho^2 U_R^2 L \qquad (26)$$

where the change in internal energy, $[\![\Gamma]\!]$, is identified as a material property (ie. a "latent heat" of transition) and is denoted as L. Here the "rupture rate", U_R, is defined as

$$U_R \equiv (\mathbf{U} - \mathbf{v}) \cdot \mathbf{n}$$

and expresses the excess of the expansion rate of the failure surface over the particle velocity in the direction normal to the surface.

Equation (26) expresses the fact that an energy barrier to failure exists, and this barrier is represented by the internal energy change $[\![\Gamma]\!]$ (or L) required for failure to occur in the material. Further, the equation shows that the rupture rate is proportional to the magnitude of the traction change (the "stress drop") divided by square root of this energy factor, that is:

$$U_R = \frac{1}{\rho} \left[\frac{[\![\tau_k \tau_k]\!]}{2L} \right]^{1/2}$$

Since ρ and L are material constants, then this says that the magnitude of the traction or stress "drop", $|\Delta\tau| \equiv [\![\tau_k \tau_k]\!]^{1/2}$, is directly proportional to the rupture rate and vice-versa. This is a dynamical condition that will reflect itself in the proper solutions for radiation fields from failure processes in different material types.

The two dynamical boundary conditions on the failure surface can be combined so as to eliminate U_R and produce a single boundary condition involving only the dynamic field variables and the intrinsic internal energy change, represented by the (material) parameter L. That is from (25) and (26) we have:

$$[\![v_k]\!] = -\left[2L / [\![\tau_k \tau_k]\!]\right]^{1/2} [\![\tau_k]\!] \qquad (27)$$

assuming that none of the traction jumps vanish (in which case the conservation relations are trivially satisfied.)

Clearly this relation shows that the jump, or discontinuity, in the particle velocity vector across the failure boundary is in the opposite direction as the jump in the traction vector (and vice-versa) and is, as well, proportional to the square root of the material parameter L. However, the relationship between particle velocity and traction jumps is non-linear, due to the dependence on the traction ratio on the right side of (27). Thus, while (25) expresses a similar working relationship between the traction and velocity jumps across the failure surface, the true non-linear relationship between these field variables is hidden in the factor involving the rupture rate. In particular, since the rupture rate is required to be proportional to the stress drop factor $|\Delta\tau|$ in order that energy be conserved, it follows that (25) represents a non-linear relationship between the traction and particle velocity field variables.

It is worth noting that the dynamics of the failure process requires that *both* the particle velocity and the tractions be discontinuous across the failure boundary. This is *not* the usual assumption made in the construction of pure kinematical models of failure, such as in a dislocation model of an earthquake where the tractions are assumed to be continuous while components of the particle velocity (and displacement) are assumed discontinuous. Neither is it the case for the variety of so called stress pulse models, where a stress drop on the failure surface is used to "drive" the surrounding (elastic) medium and thereby produce a simulated earthquake radiation field. Obviously when (current) dislocation or stress drop models are used to generate a moment representation, for purposes of inverting observational data from earthquakes, the result will be non-physical in the sense that the boundary conditions assumed are not compatible with the dynamical constraints imposed by energy and momentum conservation. Therefore "displacement offsets" or "stress drops" inferred from observations obtained through the use of such formulations do not actually represent these physical quantities. As is shown in a later section (VIII), the required form of the Green's function *surface integral representation* describing the seismic radiation involves both traction and displacement changes on the failure surface. Therefore a combination of a "stress pulse" and "dislocation" sources is generally required for compatibility with the dynamical constraints involved in spontaneous failure.

VI. The Green's Function Representation of Dynamic Fields Generated by Time Dependent Equilibrium Changes

A compact representation of the dynamic field $\mathbf{u}^{(d)}$ appearing in the equations of motion (23), which also incorporates the effects of the boundary conditions in (25) and (26), is obtained by introducing the usual Green's function for the elastic medium V_1.

Specifically, consider the Green's function $G_k^m(\mathbf{r}, t; \mathbf{r}_1, t_1)$ satisfying the inhomogeneous elastic wave equation:

$$\partial_{t_1}(\rho\, \partial_{t_1} G_k^m) - \partial_l \mathcal{G}_{lk}^m = 4\pi\, \delta_k^m\, \delta(\mathbf{r} - \mathbf{r}_1)\delta(t - t_1) \equiv 4\pi\Delta_k^m \tag{28}$$

where G_k^m is a two point tensor obeying the usual causality relations (see Archambeau and Minster, 1978, for details) and where $\mathbf{r}_1$ and t_1 are source coordinates and time, while $\mathbf{r}$ and t are receiver coordinates and time. Here δ_k^m is the Kronecker delta (which is zero if $k \neq m$ and unity if $k = m$), while $\delta(\mathbf{r} - \mathbf{r}_1)$ and $\delta(t - t_1)$ are Dirac delta functions. All partial derivatives in (28) are to be taken with respect to the *source* coordinates. Further, $\mathcal{G}_{lk}^m$ denotes the stress tensor associated with G_k^m. That is:

$$\mathcal{G}_{kl}^m = C_{ijkl}\ \partial_i G_j^m$$

where C_{ijkl} is the fourth order elastic tensor.

We can take G_k^m to satisfy boundary conditions along the exterior surface of the medium and along interior surfaces of material discontinuity, excluding the (growing) failure surface, since in the cases of most interest explicit representations of G_k^m in layered media are known. That is, G_k^m may be taken to satisfy all "normal" boundary conditions within flat or spherically layered media *except* those "non-standard" conditions, given by (25) and (26), which apply to the failure boundary. An explicit form for G_k^m which satisfies the normal material boundary conditions involving continuity of tractions and the normal component of the particle velocity (or all particle displacement components if the material boundary is a "welded" solid-solid interface) is, in the frequency domain:

$$G_l^m(\mathbf{r}, \omega; \mathbf{r}_1, \omega_1) = 4\pi \sum_{k(\omega)} \frac{\psi_m(\mathbf{r}, \omega)\ \bar{\psi}_l(\mathbf{r}_1, \omega_1)}{(\omega^2 - \omega_1^2)N(k, \omega)} \tag{29}$$

where ψ_m are the component displacement eigenfunctions for the medium (*without* the failure zone) and $k(\omega)$ denotes the infinite set of eigenvalues for the medium. (Here $\bar{\psi}_l$ is the complex conjugate of ψ_l and N (k, ω) is a normalization constant.) Since the eigenfunctions ψ satisfy the usual material boundary conditions by definition, then G_l^m also does. For the eigenfunctions in layered spheres and half spaces see, for example, Ben Menahem and Singh (1972, 1981) or Harvey (1981). (For a layered half space, however, the sum over the wave number $k(\omega)$ in the eigenfunction expansion for G_l^m may be replaced in part, or totally, by an integral, so the summation in (29) should be interpreted as a generalized sum.) These simple "layered media" cases are those of most interest in geophysics and in any case are sufficient for a study of source radiation effects.

Therefore, with the Green's function of known form satisfying all boundary conditions, except those on the failure boundary surface itself, we can obtain an integral representation of the dynamic displacement, $u_k^{(d)}$, in equation (23.) by the usual methods. That is, forming the inner product of each term in (23.) with G_k^m and integrating over the *source coordinates* in the volume V_1 *outside* the failure surface and over the *source time*, gives:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1} \partial_{t_1}\left[\rho\, \partial_{t_1} u_k^{(d)}\right] G_k^m dV_1 - \int_{-\infty}^{+\infty} dt_1 \int_{V_1} \partial_l \tau_{lk}^{(d)} G_k^m dV_1 = -\int_{-\infty}^{+\infty} dt_1 \int_{V_1} \left[\rho\ \partial_{t_1} u_k^{(e)\prime} - \rho\, g_k^{(d)}\right] dV_1$$

Likewise, forming the inner product of each term in equation (28) with $u_k^{(d)}$ and similarly integrating over the source variables gives:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1} \partial_{t_1}\left[\rho\ \partial_{t_1} G_k^m\right] u_k^{(d)} dV_1 - \int_{-\infty}^{+\infty} dt_1 \int_{V_1} \partial_l \mathcal{G}_{lk}^m\ u_k^{(d)} dV_1 = 4\pi u_m^{(d)}(\mathbf{r}, t)$$

where the formal integral properties of the delta functions on the right side of (28) have been used. Now, subtracting the second of these equations from the first and making use of the fact that

$$\tau_{lk}^{(d)} \partial_l G_k^m = \mathcal{G}_{lk}^m\ \partial_l u_k^{(d)} \quad ,$$

ultimately gives:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \frac{\partial}{\partial t_1}\left[\rho\,\frac{\partial u_k^{(d)}}{\partial t_1}\,G_k^m - \rho\,\frac{\partial G_k^m}{\partial t_1}\,u_k^{(d)}\right] dV_1 - \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \frac{\partial}{\partial x_l^{(1)}}\left[\tau_{lk}^{(d)} G_k^m - \mathcal{G}_{lk}^{m}\, u_k^{(d)}\right] dV_1$$

$$+ \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\partial_{t_1}(\rho\,\partial_{t_1} u_k^{(e)\prime}) - \rho\, g_k^{(d)}\right] G_k^m dV_1 = -4\pi u_m^{(d)}(\mathbf{r}, t) \tag{30}$$

The first integral term can be transformed so that the time integration can, in part, be evaluated. However, it is clear that the failure zone is expanding with time and so the volume V_1 exterior to this zone is time dependent. Thus we can't simply commute the time derivative with the volume integration in the first integral term, but must take account of the time variation of the spatial integration. To do so we can make use of the transport theorem for the case of a boundary that moves at a rate that is not equal to the particle velocity in the medium.

This case has been treated by several authors, for example by Eringen (1975) and by Minster (1974), the latter giving several detailed derivations of the result. (Archambeau and Minster, 1978, use the theorem in a similar application as well.) Specifically, for any function F that is dependent on the deformation within the medium (ie. dependent on the Eulerian coordinates) we have:

$$\frac{d}{dt_1}\int_{V_1(t_1)} F\, dV_1 = \int_{V_1(t_1)} \frac{\partial F}{\partial t_1}\, dV_1 + \int_{\partial V_1(t_1)} F\, \mathbf{U}\cdot\mathbf{n}\, d\mathcal{a}_1 \tag{31}$$

where $\mathbf{U}$ is the velocity vector of the boundary surface and $\mathbf{n}$ is the normal to that surface.

Now if we take the integrand in the first integral in (30) to be the function F, then we can apply (31) directly. We therefore have, for this first integral term:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \frac{\partial}{\partial t_1}\left[\rho\frac{\partial u_k^{(d)}}{\partial t_1} G_k^m - \rho\frac{\partial G_k^m}{\partial t_1} u_k^{(d)}\right] dV_1 = \int_{-\infty}^{+\infty} dt_1 \left\{ \frac{d}{dt_1}\int_{V_1(t_1)} \rho\left[\frac{\partial u_k^{(d)}}{\partial t_1} G_k^m - \frac{\partial G_k^m}{\partial t_1} u_k^{(d)}\right] dV_1 \right\}$$

$$- \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[\rho\frac{\partial u_k^{(d)}}{\partial t_1} G_k^m - \rho\frac{\partial G_k^m}{\partial t_1} u_k^{(d)}\right] U_l n_l\, d\mathcal{a}_1$$

However the first integral on the right side of this equation vanishes identically, that is:

$$\int_{-\infty}^{+\infty} dt_1 \left\{ \frac{d}{dt_1} \int_{V_1(t_1)} \left[\rho \frac{\partial u_k^{(d)}}{\partial t_1} G_k^m - \rho \frac{\partial G_k^m}{\partial t_1} u_k^{(d)} \right] dV_1 \right\} = \left\{ \int_{V_1(t_1)} \rho \frac{\partial u_k^{(d)}}{\partial t_1} G_k^m dV_1 \right]_{-\infty}^{+\infty} - \left\{ \int_{V_1(t_1)} \rho \frac{\partial G_k^{(m)}}{\partial t_1} u_k^{(d)} dV_1 \right]_{-\infty}^{+\infty} = 0$$

This follows from the fact that the medium is at rest initially so:

$$u_k^{(d)} = \frac{\partial u_k^{(d)}}{\partial t_1} = 0, \text{ as } t_1 \to -\infty$$

and since G_k^m is causal, so that

$$G_k^m(\mathbf{r}, t; \mathbf{r}, t_1) \mid_{t_1 = \infty} = \frac{\partial G_k^m(\mathbf{r}, t; \mathbf{r}, t_1)}{\partial t_1} \mid_{t_1 = \infty} = 0$$

(Actually the above is true for all $t<t_1$, but, in particular, when t_1 becomes infinite then it is true for all receiver times t.)

Therefore, using these results in (30) and further applying the ordinary form of Gauss' Theorem to the second integral in (30), gives:

$$4\pi u_m^{(d)}(\mathbf{r}, t) = \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left\{ \left[\tau_{lk}^{(d)} + \rho \frac{\partial u_k^{(d)}}{\partial t_1} U_l \right] G_k^m - \left[G_{lk}^m + \rho \frac{\partial G_k^m}{\partial t_1} U_l \right] u_k^{(d)} \right\} n_l d a_1$$

$$- \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\frac{\partial}{\partial t_1} \left(\rho \frac{\partial u_k^{(e)\prime}}{\partial t_1} \right) - \rho\, g_k^{(d)} \right] G_k^m dV_1 \qquad (32)$$

This result is one form of the integral Green's function representation of the dynamic radiation field, $u_m^{(d)}$, produced by a growing failure zone in an initially stressed medium. The result is formally exact.

The first integral term on the right side of (32) involves an integration over the failure boundary and the integrand in brackets contains the same combination of terms as are involved in the conservation relation (25). Thus, the integral representation contains the natural boundary conditions that apply to the physical process. Since only dynamical field quantities are involved in this surface integral term, it is appropriate to interpret this integral as representing the interaction of the dynamic field with the (growing) failure boundary. As such it would represent scattering and, possibly, absorption effects at the boundary. (Note that the rupture rate, $U_R \equiv U_l n_l$, appears in the integrand of the integral and that U_R is a function of the energy, L, absorbed in the process of failure.)

The second volume integral contains the inertial term arising from changes in the equilibrium field in the elastic medium surrounding the failure zone and corresponds to the fundamental source term that gives rise to the dynamical response of the medium. Because this term generally represents a relaxation of stress and a reduction of stored strain energy (which shows up as energy in the radiated elastic wave field) the elastic waves produced by this source term have been called *relaxation source fields* (Archambeau, 1964, 1968) and used to approximate earthquake radiation fields. This term represents a release of energy from the entire prestressed medium surrounding the failure zone and, as such, is the fundamental source of the elastic wave radiation produced by failure. In this regard the gravity term appearing with the relaxation source term involves the dynamic changes in the gravity field due to density changes in the medium. As noted earlier and as described in the Appendix 1, it is very small compared to the relaxation term and can be neglected.

VII. Equivalences: Initial Value (Relaxation) Sources

As was just mentioned, the result given in (32) is slightly different than that obtained in earlier work using a different method of derivation, which was based on the view that the radiation process associated with failure in a stressed medium could be described as an initial value problem. Nevertheless the representation in (32) can be transformed to a form that is nearly identical to that obtained from the earlier analysis.

In order to show this latter "equivalence", consider the integral term involving the inertial source factor ($\partial_{t_1}(\rho\, \partial_{t_1} u_k^{(e)'}$). We note that the integrand in this integral may be "expanded" to the form:

$$\left[\partial_{t_1}\left(\rho\, \partial_{t_1} u_k^{(e)'}\right)\right] G_k^m = \partial_{t_1}\left[\rho\, \partial_{t_1} u_k^{(e)'} (G_k^m)\right] - \rho\, \partial_{t_1} u_k^{(e)'} (\partial_{t_1} G_k^m)$$

and that the right side of this identity can be used in the integral instead of the quantity on the left. Thus the integral term in question can be written as:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \partial_{t_1}\left(\rho\, \partial_{t_1} u_k^{(e)'}\right) G_k^m dV_1 = \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \rho\, \partial_{t_1} u_k^{(e)'}\, \partial_{t_1} G_k^m dV_1 - \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \partial_{t_1}\left[\rho\, \partial_{t_1} u_k^{(e)'} \left(G_k^m\right)\right] dV_1$$

(33)

The first integral term on the right side of this equation is that obtained as the "relaxation source" term in the initial value formulation, while the second integral is an extra term not explicitly present in the initial value formulation results. This latter term can, however, be recast into the form that can be combined with the surface integral terms in (32) by using the transport theorem result in (31). That is, using $\rho\, \partial_{t_1} u_k^{(e)'} G_k^m$ for F in (31) gives:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \partial_{t_1}\left[\rho\, \partial_{t_1} u_k^{(e)'} (G_k^m)\right] dV_1 = \int_{-\infty}^{+\infty} dt_1 \left[\frac{d}{dt_1} \int_{V_1(t_1)} \rho\, \partial_{t_1} u_k^{(e)'} G_k^m dV_1\right] - \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[\rho\, \partial_{t_1} u_k^{(e)'} G_k^m\right] U_l n_l d\mathcal{a}_1$$

(34)

The time integration in the first integral on the right yields the spatial volume integral evaluated at the (infinite) time limits. Therefore the first integral on the right is:

$$\int_{-\infty}^{+\infty} dt_1 \left[\frac{d}{dt_1} \int_{V_1(t_1)} \rho\, \partial_{t_1} u_k^{(e)'} G_k^m dV_1\right] = \left[\int_{V_1(t_1)} \rho\, \partial_{t_1} u_k^{(e)'} G_k^m dV_1\right]_{-\infty}^{+\infty} = 0 \tag{35}$$

where the integral vanishes since the equilibrium field is taken to change continuously (the failure zone growth described by the rupture rate U_R is taken to be a continuous function of time) and since:

$$\lim_{t_1 \to -\infty} \partial_{t_1} u_k^{(e)'} = 0 \; ; \; \lim_{t_1 \to +\infty} G_k^m(\mathbf{r}, t; \mathbf{r}_1, t_1) = 0$$

Thus, using (35) and (34) in the equation (33) gives:

$$-\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \partial_{t_1}\left[\rho\, \partial_{t_1} u_k^{(e)'}\right] G_k^m dV_1 = \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \rho\, \partial_{t_1} u_k^{(e)'}\, \partial_{t_1} G_k^m dV_1 + \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[\rho\, \partial_{t_1} u_k^{(e)'} G_k^m\right] U_l n_l d\mathcal{a}_1$$

(36)

Inserting this result in the basic representation integral for the dynamic field, equation (32), therefore produces:

$$4\pi u'_m(\mathbf{r}, t) = 4\pi u_m^{(e)'} + \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left\{ \left[\tau_{lk}^{(d)} + \rho \frac{\partial u'_k}{\partial t_1} U_l\right] G_k^m - \left[G_{lk}^m + \rho \frac{\partial G_k^m}{\partial t_1} U_l\right] u_k^{(d)} \right\} n_l d\mathcal{a}_1$$

$$+\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \rho \frac{\partial u_k^{(e)'}}{\partial t_1}\left[\frac{\partial G_k^m}{\partial t_1}\right] dV_1 + \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \rho g_k^{(d)} G_k^m dV_1 \tag{37}$$

where a factor $4\pi u_m^{(e)'}$ has been added to both sides of the result in order to put the final equation in a form comparable to that given by Archambeau and Minster (1978). In this result we have defined relative displacements, measured from the initial state, so that as previously defined

$$\mathbf{u}^{(e)'} = \mathbf{u}^{(e)} - \mathbf{u}^{(I)}$$

for the equilibrium fields, and similarly

$$\mathbf{u}' = \mathbf{u} - \mathbf{u}^{(I)}$$

for the "total" fields (dynamic plus equilibrium fields). Thus relative fields are measured from the fixed initial equilibrium state (with displacement $\mathbf{u}^{(I)}$) and denoted by primes throughout. In view of these definitions and the relation $\mathbf{u} = \mathbf{u}^{(d)} + \mathbf{u}^{(e)}$, then

$$\mathbf{u}' = \mathbf{u}^{(d)} + \mathbf{u}^{(e)'}$$

If the start and completion of the failure zone is taken to be the time interval $(0, \tau)$, then the interval of time integrations appearing in (37) can be reduced. That is, $U_l n_l = 0$ and $\partial_{t_1} u_k^{(e)} = 0$ for $t_1<0$ and for $t_1>\tau$ in the integrals, and therefore the time limits on integrals involving these factors may be reduced accordingly. In particular, they become the interval $(0, \min[t, \tau])$, as noted by Archambeau and Minster (1978). With the convention that the failure zone development occurs over the time interval $(0, \tau)$, then we also have that $\mathbf{u}^{(e)'} = \mathbf{u}^{(I)} - \mathbf{u}^{(I)} = 0$, for $t_1 < 0$; while $\mathbf{u}^{(e)'} = \mathbf{u}^{(F)} - \mathbf{u}^{(I)}$, for $t_1 > \tau$, with $\mathbf{u}^{(F)}$ denoting the final equilibrium field.

With the conventions of a specific reference state and a finite source time interval from 0 to τ (which involve no loss of generality), we get similar results from (37) as those given by Archambeau and Minster. In particular, the identical volume integrals corresponding to relaxation effects and gravitational effects. However, the surface integral in (37), while of the same form as that obtained from the initial value formulation, is nevertheless different in detail than that obtained earlier. In particular, the dynamic stresses and displacements, $\tau_{lk}^{(d)} \equiv \tau_{lk} - \tau_{lk}^{(e)}$ and $u_k^{(d)} \equiv u_k - u_k^{(e)}$, appear in the surface integral in (37) while in the initial value formulation these are (implicitly) replaced by the "relative" stress and displacement components $\tau'_{lk} \equiv \tau_{lk} - \tau_{lk}^{(I)}$ and $u'_k \equiv u_k - u_k^{(I)}$. The fields $\tau_{lk}^{(e)}$ and $u_k^{(e)}$ are parametrically time varying during the failure process (by definition) and the fields $\tau_{lk}^{(I)}$ and $u_k^{(I)}$ are fixed, *time independent*, values of equilibrium stress and displacement before initiation of the failure process.

Considering the simplicity and rigor of the derivations leading to the representations in (32) and subsequently to (37), it is concluded that (37) is a proper exact representation of an initial value formulation of a spontaneous failure source (earthquake). Thus it appears that the direct initial value formulation obtained earlier did not express the surface integral term completely, or at least that this latter formulation was imprecise as to the definitions of the field variables in the integral terms.

Since the surface integral term represents scattering and absorption of energy at the failure zone boundary, it may often produce smaller second order effects in the radiation field compared to the relaxation integral term in (32) or (37). Indeed, the early models generated from the initial value formulation entirely neglected the surface integral term, along with the very small gravity effects, so that only the relaxation term in (37) was used to represent the radiation. (The resulting field predictions were called "transparent source approximations".) Thus, the earlier source models never included the surface integral contributions, so that in the use of these approximate models the exact nature of contributions from the surface integral term never arose.. (However, neglect of the surface integral contribution was done without analytical justification, with only intuitive arguments used). The representations in (32) or (37) can, in any case, be used to accurately determine the nature and size of the true scattering-absorption effects.

Some concrete justification for neglect of the "scattering-absorption" term has, however, been provided by Stevens (1981) who compared the "transparent source approximation" for a spherical transition zone in a prestressed medium with the exact solution for the same problem; with the latter containing the proper surface integral scattering term. This comparison showed that the surface integral did indeed contribute small reflection-refraction effects, but that a much larger direct first arrival radiation field was obtained from the relaxation integral term and that it was a good approximation to the total source radiation field. However, since the source considered by Stevens was an instantaneously created spherical failure zone, it did not contain the possibilities for absorption of energy as would a spontaneous failure process. Thus the "absorption" effects at a growing boundary were not included in the comparison. While these latter effects might in fact "damp out" scattering from the source boundary, so that the two effects might cancel each other out in part, it cannot be confidently concluded that this would always be the case. That is, it is not clear that absorption is of the same order as scattering in its effect on the total radiation field and could, at least in some circumstances and at some points on the failure surface, be a considerably larger effect and so significantly modify the total field from that predicted by the relaxation term alone. (For example this might well be the case for radiation contributions from near the front of an advancing thin ellipsoidally shaped failure zone).

Therefore, given all these possibilities, retention of the surface integral contributions in specific source models seems desirable in most cases of spontaneous failure. Ideally then, models of earthquakes should include the scattering-absorption surface integral as well as the relaxation term, whether (32) or (37) is used as a basis for the model predictions.

It is also appropriate to note that the earlier initial value development employed the device of considering the rupture process as being a series of elemental discontinuous changes in the rupture dimensions, with these changes producing corresponding discontinuous changes in the equilibrium field $u_k^{(e)}$. By taking these changes to be of infinitesimal size, and the time interval between them to be of infinitesimal duration, a limiting process of summing the infinitesimal contributions was used to produce the effects of continuous failure growth. By contrast, the present approach treats rupture growth and changes in the equilibrium field to be continuous from the beginning. These two methods should give the same results, provided the failure process proceeds in a continuous fashion. If, however, there is a finite discontinuous change in the rupture rate and a corresponding discontinuity in the equilibrium field, then, as noted by Stevens (1980, 1981), the integral in equation (35) will not be zero. However, for earthquakes which result from spontaneous rapid failure resulting from slow loading, it is not likely that discontinuous changes in the rupture rate occur, so that the imposition of a continuous time varying rupture rate in the formulation seems justified. Nevertheless, discontinuous changes cannot be ruled out with absolute certainty. Such cases can, however, be considered in either formulation provided care is taken to explicitly account for any finite and instantaneous changes in the rupture rate and the equilibrium field $u_k^{(e)}$.

An example of a singular case in which a "supersonic" rupture rate combined with spherical symmetry of the developing failure zone produces discontinuous behavior occurs when an explosion is detonated in a prestressed medium. In this case the failure zone can be completely formed by a supersonic shock wave before the medium outside this failure zone can react dynamically, since the shock velocity (and rupture rate) is taken to be larger, at all points on the spherical failure surface, than the highest intrinsic velocity of signal propagation in the medium. Consequently the failure zone is, in essence, formed instantaneously in-so-far as the (causal) relaxation of the equilibrium field external to this failure zone in concerned. This type of source has been treated in detail by Archambeau (1968, 1972) as an initial value problem, in the same spirit as in the case of spontaneous failure, but where the discontinuous change in the equilibrium field at the conclusion of the shock induced failure process is accounted for explicitly from the beginning. Stevens (1980, 1981) obtains the same result using a method similar to the "time dependent equilibrium method" described here, but tailored to the case of the instantaneous spherical failure zone.

In the present formulation, if we impose the conditions of this singular problem on the integral representation in either of the equivalent forms given by (32) and (37) we get the same results obtained by Archambeau and Stevens; even though the representations in (32) and (37) were derived under the assumptions of continuous failure zone growth with a finite rupture rate. In particular, taking the time at which the source is initiated (ie. $t_1 = 0$) as being the time at which the rupture zone has been formed, so that $U_l n_l \equiv U_R = 0$ for all times equal or greater than $t_1 = 0$, then:

$$\mathbf{u}^{(e)\prime}(\mathbf{r}, t_1) = \begin{cases} 0 \; ; t_1 < 0 \\ \left[\mathbf{u}^{(F)} - \mathbf{u}^{(I)}\right] H(t_1) \; ; \; t_1 \geq 0 \end{cases}$$

where H (t_1) is a step function in time centered at $t_1 = 0$. Now both (32) and (37) give:

$$4\pi u_m^{(d)}(\mathbf{r}, t) = \int_0^{+\infty} dt_1 \int_{\partial V_1} \left[\tau_{lk}^{(d)} G_k^m - \mathcal{G}_{lk}^m u_k^{(d)}\right] n_l d\mathcal{a}_1 + \int_{V_1} \rho u_k^* \left[\frac{\partial G_k^m}{\partial t_1}\right] dV_1 \tag{38}$$

where $u_k^* \equiv \left[u_k^{(F)} - u_k^{(I)}\right]$ and where V_1 is the fixed volume exterior to the failure zone. This is the same result obtained when the discontinuous effects are explicitly accounted for when deriving the integral representation. Thus the results in (32) and (37) apply to this discontinuous case as well as to all continuous rupture cases.

VIII. Equivalences: "Dislocation-Stress Pulse" Sources

Another form of equivalent representation of the radiation field due to failure in a stressed medium can be derived from (32) and this representation involves only integrals over the failure boundary. Therefore it can be compared to kinematical representations (dislocation models) and to the so-called stress pulse representations that involve only integrals over the failure surface. The approach followed is a simple generalization of one used by Stevens (1981), which was introduced to demonstrate a similar equivalence when a failure zone is "instantaneously" created. In this case we want a result similar to Stevens' result, but for the general case of a finite rupture rate.

To obtain the desired representation we need to transform the relaxation integral term in (32) to a surface integral taken over the failure boundary. To do so we will need to develop a purely formal integral identity involving the equilibrium field $u_k^{(e)\prime}$ and the dynamical Greens function G_k^m. This is achieved by considering the basic differential equations satisfied by these fields, which are:

$$\partial_l \tau_{lk}^{(e)'} = -\rho\, g_k^{(e)'}$$

$$\partial_l G_{lk}^{m} - \partial_{t_1}(\rho\, \partial_{t_1} G_k^m) = -4\pi\Delta_k^m(\mathbf{r}-\mathbf{r}_1;\, t-t_1)$$

Taking the inner product of the equation for G_k^m with $u_k^{(e)'}$ on both sides, and integrating over the source time and spatial coordinates, gives:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\partial_{t_1}(\rho\, \partial_{t_1} G_k^m)\right] u_k^{(e)'} dV_1 - \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \partial_l G_{lk}^{m}\, u_k^{(e)'} dV_1 = 4\pi \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \Delta_k^m u_k^{(e)'} dV_1 = 4\pi u_m^{(e)'}$$

Now taking the inner product of the first equation with G_k^m on both sides, integrating and subtracting the result from the previous equation, gives:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\partial_{t_1}(\rho\, \partial_{t_1} G_k^m)\right] u_k^{(e)'} dV_1 - \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\partial_l G_{lk}^{m}\, u_k^{(e)'} - \partial_l \tau_{lk}^{(e)'} G_k^m\right] dV_1 \qquad (39)$$

$$+ \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\rho\, g_k^{(e)'}\right] G_k^m dV_1 = 4\pi u_m^{(e)'}$$

Noting that

$$G_{lk}^{m}\, \partial_l u_k^{(e)'} = \tau_{lk}^{(e)'}\, \partial_l G_k^m\, ,$$

due to the symmetry of the elastic tensor C_{ijkl}, then it follows that:

$$\partial_l G_{lk}^{m}\, u_k^{(e)'} - \partial_l \tau_{lk}^{(e)'} G_k^m = \partial_l\left[G_{lk}^{m}\, u_k^{(e)'} - \tau_{lk}^{(e)'}\, G_k^m\right]$$

Thus the integrand in the second integral on the left in (39) can be replaced with the divergence term in the equation above and in this form the resulting volume integral can be transformed to a surface integral using Gauss' theorem. We therefore obtain from (39):

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\partial_{t_1}(\rho\, \partial_{t_1} G_k^m)\right] u_k^{(e)'} dV_1 = 4\pi u_m^{(e)'} + \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[G_{lk}^{m}\, u_k^{(e)'} - \tau_{lk}^{(e)'} G_k^m\right] n_l\, d\mathcal{A}_1 \qquad (40)$$

$$- \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\rho\, g_k^{(e)'}\right] G_k^m dV_1$$

However, we note that

$$\left[\partial_{t_1}(\rho\, \partial_{t_1} G_k^m)\right] u_k^{(e)'} - \left[\partial_{t_1}(\rho\, \partial_{t_1} u_k^{(e)'})\right] G_k^m = \partial_{t_1}\left[\rho\, \partial_{t_1} G_k^m u_k^{(e)'} - \rho\, \partial_{t_1} u_k^{(e)'} G_k^m\right]$$

Therefore we can replace the volume integral term on the left side of (40) using this last equation. This produces:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\partial_{t_1}\left[\rho\, \partial_{t_1} u_k^{(e)'}\right]\right] G_k^m dV_1 = 4\pi u_m^{(e)'} + \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[G_{lk}^m u_k^{(e)'} - \tau_{lk}^{(e)'} G_k^m\right] dV_1$$
$$- \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \frac{\partial}{\partial t_1}\left[(\rho\, \partial_{t_1} G_k^m)\, u_k^{(e)'} - (\rho\, \partial_{t_1} u_k^{(e)'})\, G_k^m\right] dV_1 - \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \rho\, g_k^{(e)'} G_k^m dV_1$$

In addition, we can apply the transport theorem to the volume integral involving the time derivative on the right side of this equation to transform it to a surface integral. That is, using the transport theorem given in equation (31), we get:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \frac{\partial}{\partial t_1}\left[(\rho\, \partial_{t_1} G_k^m) u_k^{(e)'} - (\rho\, \partial_{t_1} u_k^{(e)'}) G_k^m\right] dV_1 = \left\{\int_{V_1(t_1)} \left[(\rho\, \partial_{t_1} G_k^m) u_k^{(e)'} - (\rho\, \partial_{t_1} u_k^{(e)'}) G_k^m\right] dV_1\right\}_{-\infty}^{+\infty}$$
$$- \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[\left[\rho\, \partial_{t_1} G_k^m\right] u_k^{(e)'} - \left[\rho\, \partial_{t_1} u_k^{(e)'}\right] G_k^m\right] U_l n_l d\mathcal{a}_1$$

where the first term on the right vanishes at the infinite limits; since $\partial_{t_1} G_k^m = G_k^m = 0$, for $t_1 \to +\infty$; and $u_k^{(e)'} = \partial_{t_1} u_k^{(e)'} = 0$, for $t_1 \to -\infty$. Therefore the previous equation can be rewritten in the final desired form as:

$$\int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \left[\partial_{t_1}\left[\rho\, \partial_{t_1} u_k^{(e)'}\right]\right] G_k^m dV_1 = 4\pi u_m^{(e)'} + \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left\{\left[G_{lk}^m + \rho\, U_l\, \partial_{t_1} G_k^m\right] u_k^{(e)'} - \left[\tau_{lk}^{(e)'} + \rho\, U_l\, \partial_{t_1} u_k^{(e)'}\right] G_k^m\right\} n_l d\mathcal{a}_1$$
$$- \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \rho\, g_k^{(e)'} G_k^m dV_1 \qquad (41)$$

This integral identity now expresses the relaxation term that appears in (32) in terms of a surface integral on the failure boundary. Thus (41) can be used to transform the basic representation integral equation in

(32) into the following equivalent representation:

$$4\pi u'_m(\mathbf{r}, t) = \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left\{ \left[\tau'_{lk} + (\rho\, \partial_{t_1} u'_k) U_l \right] G_k^m - \left[\mathcal{G}_{lk}^m + (\rho\, \partial_{t_1} G_k^m) U_l \right] u'_k \right\} n_l\, d\mathcal{a}_1$$

$$+ \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \rho\, g'_k G_k^m dV_1 \qquad (42)$$

where we have a result expressed in terms of the "relative" displacement, stress and gravity fields; that is $u'_k = u_k^{(d)} + u_k^{(e)'}$, etc.

Equation (42) has a form that is quite similar to the standard elastodynamic integral equation used to obtain equivalent dislocation "models" for earthquakes. (If $U_l n_l \equiv U_R$ were set to zero in (42), then it would have the identical form.) However, the integral representation in (42) is actually considerably more complex than the standard form; in spite of its relatively simple expression.

In order to display this complexity and isolate the various contributions to the radiation field, it is better to rewrite (42) with the dynamic and equilibrium field explicitly displayed. That is, (42) can be rewritten in expanded form as:

$$4\pi u'_m(\mathbf{r}, t) = \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left\{ \left[\tau_{lk}^{(d)} + (\rho\, \partial_{t_1} u_k^{(d)}) U_l \right] G_k^m - \left[\mathcal{G}_{lk}^m + (\rho\, \partial_{t_1} G_k^m) U_l \right] u_k^{(d)} \right\} n_l\, d\mathcal{a}_1$$

$$+ \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left\{ \left[\tau_{lk}^{(e)'} + (\rho\, \partial_{t_1} u_k^{(e)'}) U_l \right] G_k^m - \left[\mathcal{G}_{lk}^m + (\rho\, \partial_{t_1} G_k^m) U_l \right] u_k^{(e)'} \right\} n_l d\mathcal{a}_1 + \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} (\rho\, g'_k)\, G_k^m\, dV_1 \quad (43)$$

where the (time varying) equilibrium field contributions are explicitly displayed along with the purely dynamical parts of the displacement-stress fields. Here, of course, $u'_m = u_m^{(d)} + u_m^{(e)'}$ on the left hand side of the equation, so that the relative displacement field is represented.

The boundary conditions at the failure surface are such that the rupture rate vector **U** is a function of both the equilibrium and dynamic fields (see equation 26). Further, from equations (18) and (25), we have the boundary conditions:

$$\left.\begin{aligned}\left[\!\left[\tau_{lk}^{(e)}\, n_l\right]\!\right] &= 0 \quad ; \quad \mathbf{r}\, \varepsilon\, \partial V_1 \\ \rho U_R \left[\!\left[v_k\right]\!\right] &= -\left[\!\left[t_k\right]\!\right] \quad ; \quad \mathbf{r}\, \varepsilon\, \partial V_1\end{aligned}\right\} \tag{44}$$

with $U_R = U_l n_l$, corresponding to the rupture rate. Here v_k and $t_k = \tau_{lk} n_l$ are the total velocity and traction vector components, involving the sum of equilibrium and dynamic fields. Consequently, using these conditions together gives:

$$\rho U_R \left[\!\left[v_k\right]\!\right] = -\left[\!\left[t_k^{(d)}\right]\!\right] \tag{45a}$$

Thus, on the failure boundary $\partial V_1\,(t_1)$:

$$\rho U_R v'_k + t_k^{(d)} = \rho U_R\, {}^{(1)}v'_k + {}^{(1)}t_k^{(d)} \tag{45b}$$

where ${}^{(1)}v'_k$ and ${}^{(1)}t_k^{(d)}$ denote field variables from within the failure zone. Also, the velocity fields both inside and outside the failure zone have the form $v'_k = v_k^{(d)} + v_k^{(e)'}$. Thus the boundary condition applying on the failure surface involves both the equilibrium and dynamic velocity components in the particular combination expressed by (45). Therefore, it is appropriate to rearrange the terms in (43) so that the boundary conditions expressed in (45) apply to one of the surface integrals, while the first of the conditions in (44) applies to the other surface integral. That is, rearranging (43) to accommodate the boundary conditions in separate integral terms yields:

$$\begin{aligned}4\pi u'_m(\mathbf{r}, t) = &\int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left\{ \left[t_k^{(d)} + \rho U_R\, \partial_{t_1} u'_k\right] G_k^m - \left[g_k^m\, u_k^{(d)} + (\rho U_R\, \partial_{t_1} G_k^m)\, u'_k\right] d\mathcal{a}_1 \right. \\ &+ \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[t_k^{(e)'}\, G_k^m - g_k^m\, u_k^{(e)'}\right] d\mathcal{a}_1 + \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} (\rho\, g'_k)\, G_k^m\, dV_1\end{aligned} \tag{46}$$

where g_k^m denotes the tractions associated with the Greens function G_k^m. In this form the second surface integral involves only equilibrium field changes and the first boundary condition in (44), pertaining to the equilibrium field, applies to it. In this regard we note that the first boundary condition in (44) applies to the equilibrium traction $\tau_{lk}^{(e)}\, n_l = (\tau_{lk}^{(e)'} + \tau_{lk}^{(1)}) n_l$. But since $\left[\!\left[\tau_{lk}^{(1)}\, n_l\right]\!\right] = 0$ also, then it follows that traction continuity applies to the *relative* stresses as well. That is

$$[\![\tau_{lk}^{(e)'} n_l]\!] = 0$$

and this relation applies in the second integral. On the other hand, the first surface integral involves dynamic and equilibrium field changes at the boundary, with the boundary condition (45) applying.

If there were no equilibrium field changes, then the second integral in (46) would vanish, since the equilibrium stress and displacement fields appearing in (46) are measured relative to the initial state of the medium. In this case there could be no failure boundary growth, so $U_R = 0$ is implied, and the first integral would reduce to the standard form with V_1 and ∂V_1 fixed. Then we would have:

$$4\pi u_m^{(d)} = \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1} \left[t_k^{(d)} G_k^m - g_k^m u_k^{(d)} \right] d\mathcal{a}_1 + \int_{-\infty}^{+\infty} dt_1 \int_{V_1} \rho\, g_k^{(d)} G_k^m \, dV_1$$

and the first integral would represent pure scattering from the fixed surface ∂V_1, while the volume integral would be non-zero if there were dynamical changes in the gravity field.

Using this limiting case as a guide to the interpretation of (46), we can rewrite this equation as:

$$4\pi u'_m(r\,,t) = \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[t_k^{(d)} G_k^m - g_k^m u_k^{(d)} \right] d\mathcal{a}_1 + \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \rho\, U_R \left[(\partial_{t_1} u'_k) G_k^m - (\partial_{t_1} G_k^m) u'_k \right] d\mathcal{a}_1$$
$$+ \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[t_k^{(e)'} G_k^m - g_k^m u_k^{(e)'} \right] d\mathcal{a}_1 + \int_{-\infty}^{+\infty} dt_1 \int_{V_1(t_1)} \rho\, g'_k G_k^m \, dV_1 \qquad (47)$$

and interpret the first integral as the term due to scattering, the second integral as that due to absorption of energy along the growing boundary (because of the presence of U_R and its dependence on the energy, L, required for failure) and the third integral as being the equivalent for stress relaxation and energy release in response to the growing failure boundary. Here again, the last integral is the (generally negligible) gravity term. Because of the boundary condition in (45), the first and second integrals are actually connected - which would be expected since scattering and absorption at the failure boundary must be interconnected, as was noted earlier.

The field decomposition can be symbolically expressed as:

$$4\pi\, \mathbf{u}'(\mathbf{r}\,,t) = \left[I_{sc}\,(\mathbf{u}^{(d)}) + I_a\,(\mathbf{u}^{(d)} + \mathbf{u}^{(e)'}) \right] + I_R\,(\mathbf{u}^{(e)'}) + I_g(\mathbf{g}') \tag{48}$$

where the terms on the right denote the scattering (I_{sc}), absorption (I_a), relaxation (I_R) and gravity (I_g) terms in (47). Here only the dependence on the dynamic and/or equilibrium displacements are explicitly displayed, since the corresponding tractions appear with these displacements in the integral terms and can be derived from them. (The brackets are used to indicate that the two integral terms I_{sc} and I_a should be treated together in solving this equation.)

The relaxation term in equation (47) is now in the form of a surface integral, with the possibility that it can be viewed in terms of equivalent stress-pulse and dislocation sources. In particular, we can write this term as:

$$I_R\,(\mathbf{u}^{(e)'}) = 4\pi \left[u_m^{(S)} + u_m^{(D)} \right] \tag{49a}$$

with $u_m^{(S)}$ and $u_m^{(D)}$ as displacement fields due to equivalent stress-pulse and dislocation type sources, defined by

$$\left. \begin{aligned} 4\pi u_m^{(S)} &\equiv \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} t_k^{(e)'}\, G_k^m \, d\mathcal{a}_1 \\ 4\pi u_m^{(D)} &\equiv -\int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} u_k^{(e)'}\, g_k^m \, d\mathcal{a}_1 \end{aligned} \right\} \tag{49b}$$

These two equivalents are not independent however, since once the equilibrium displacement change $u_k^{(e)'}$ has been determined for the static inclusion problem then $t_k^{(e)'}$ may be simply derived from it. Further, for the static inclusion problem involved here, we have

$$u_k^{(e)} = u_k^{(e)'} + u_k^{(I)}$$

where $u_k^{(e)}$ and $u_k^{(I)}$ are the displacement fields with and without a failure zone, while $u_k^{(e)'}$ represents the change in the initial state due to the presence of the failure zone. Here, $u_k^{(e)}$ $u_k^{(e)\prime}$ and $u_k^{(I)}$ are all components of biharmonic vector fields. Thus, we already know the spatial form of both $u_k^{(e)}$ and $t_k^{(e)}$, and that the coefficients of the biharmonic series for $u_k^{(e)\prime}$, and the corresponding traction $t_k^{(e)\prime}$, are parametrically dependent on the source time t_1 due to the variable dimensions of the failure zone with time. Therefore the surface integrals in (49) can be analytically evaluated once the failure surface is specified.

These equivalent source integrals can be expressed in more explicitly recognizable form if we use the definition of $u_k^{(e)\prime}$ in terms of field changes from the initial state. That is, we can also write (49) as:

$$4\pi\, u_m^{(S)} \equiv \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[t_k^{(e)} - t_k^{(I)}\right] G_k^m \, d\mathcal{a}_1$$

$$4\pi\, u_m^{(D)} = \int_{-\infty}^{+\infty} dt_1 \int_{\partial V_1(t_1)} \left[u_k^{(I)} - u_k^{(e)}\right] g_k^m \, d\mathcal{a}_1 \qquad (50)$$

These two source equivalents, corresponding in sum to the relaxation source field term, may be interpreted as a continuously variable "stress-pulse" on the expanding failure surface, with magnitude equal to the traction "drop" on this surface, plus a continuously expanding closed dislocation surface with a dislocation "offset" equal to the displacement change from the initial state at the failure boundary.

Thus we find that failure induced volumetric stress relaxation and the seismic radiation associated with it can be accounted for by a superposition of two familiar surface distributed source equivalents; namely a pair of related stress-pulse and dislocation equivalents. Since the radiation fields produced by each of these equivalents will be superposed on one another, one can expect a total field having characteristics that may be quite different from either of the equivalents separately. In any case the equivalent form of the integral equation (47), or alternately as expressed in (48) and (49), constitutes a second representational form for the elastodynamic radiation produced by failure of a stressed medium and we see that this equivalent form contains not only combined stress-pulse and dislocation equivalents that account for volume relaxation effects, but boundary absorption-scattering terms that are not usually negligible. Therefore it is generally necessary to solve the integral equation in (47)-(48) to obtain the total wave radiation field. These applications are investigated in detail by Archambeau and Dilts (1989).

IX. Summary and Conclusions

Examination of some of the previously developed representations for seismic radiation from earthquakes, particularly those using the divergence of the medium stress, or the "stress glut", in the moment tensor representations of earthquake radiation fields leads to the conclusion that these representations have no rational physical basis, nor can they be logically related, in even an approximate sense, to either the kinematics or dynamics of such a source.

The appropriate representation of the radiation field can, however, be obtained in a quite straightforward manner by noting that the displacement-stress fields in the medium are a sum of dynamic and equilibrium components and that the equilibrium field component is time dependent when there is spontaneous failure within the medium. It is shown that this temporal variation in the equilibrium field, which is due to creation of a failure boundary within an initially stressed medium, gives rise to an "equivalent force" term in the equations of motion in the linear zone *outside* the failure boundary and corresponds to the *inertial effect* of changes in the *equilibrium displacement field* in the medium surrounding the failure zone. Further, it is shown that this effect only occurs when the medium is prestressed and a new boundary, enclosing a zone of material altered by the failure process, is created. This inertial term, rather than a term involving the divergence of the stress in the medium, is the proper "equivalent force term" associated with an earthquake. However, because the process of failure is spontaneous and depends on the dynamic radiation field for its continued growth, the near field radiation effects are complicated by energy absorption and failure boundary scattering that will also manifest themselves in the total radiation field observed. These latter effects are determined by the boundary conditions, expressing conservation laws, on the growing failure surface.

In order to treat both the inertial or *relaxation* effects in the medium surrounding the failure boundary and the boundary scattering and energy absorption associated with failure growth, it is appropriate to reformulate the representation of the problem as an integral equation involving both the relaxation effects and the boundary interaction effects. A Green's function integral equation for the displacement in the linear region outside the failure zone volume is therefore developed using both the equations of motion and the boundary conditions on the (growing) failure surface. The resulting integral representation of the radiation field involves a volume integral accounting for the relaxation of the equilibrium field in the elastic medium surrounding the failure zone (the dominant effect) and surface integral terms that may be individually identified as being associated with scattering and energy absorption. Further it is shown that this "representation theorem" reduces to the classical result in cases when there is no failure at all, or when the failure process has stopped.

The integral representation obtained is shown to be susceptible to transformation to other forms, which is useful from a computational point of view as well as being important from a conceptual standpoint. In particular, it is shown that the integral term representing the relaxation related radiation can be put in a form identical to that obtained in previous work (*eg*. Archambeau, 1968; Archambeau and Minster, 1978) which used an initial value approach to generate the representation of the radiation field. However, it is found that the surface integral terms representing scattering and absorption at the failure boundary are more precisely defined by the representation obtained here and have a slightly different form that represents, at the least, a clarification of the earlier results. Alternately the volume integral representing seismic radiation from relaxation in the medium surrounding the failure zone can be transformed into surface integrals over the failure zone boundary which can be identified as the superposition (or sum) of a surface distributed dislocation and a distributed stress-pulse involving *only* the time dependent changes in the *equilibrium fields* at the boundary. These two equivalents are not independent however, since they are both the result of changes in the equilibrium field from its initial state. (This latter field can be obtained by standard methods of solution for inclusion problems). The remaining surface integral terms, involving both the dynamic and equilibrium field changes, can again be individually identified as terms representing scattering and absorption at the failure boundary.

The integral equation representations describing the failure induced seismic radiation, whether those containing a volume relaxation term explicitly or that representation with this particular term represented by the dislocation-stress pulse equivalent, are exact and applicable to any prestress state and source geometry. Further, the rupture geometry and growth rate are prescribed in the integral representation when the boundary condition involving energy conservation is used.

In some cases it is likely that the surface distributed dislocation-stress pulse equivalent form will be more convenient from a computational standpoint, but in any case the relaxation integral or the equivalent dislocation-stress pulse integrals can be evaluated independently from the remainder of the integral equation if a failure zone geometry is assumed, since these integrals depend only on the equilibrium field changes which can be obtained independently through solutions of static inclusion problems. This means that a moment type expansion can be obtained for the main (relaxation term) contribution to the radiation field by standard methods and that corrections, or perturbations, to these moment terms can be obtained by iterative approximation of the scattering-absorption integrals. (These approximations are considered in detail in a companion study by Archambeau and Dilts, 1989).

Thus the integral representations obtained here form the basis for a moment-tensor representation of earthquake source that is dynamical in nature and satisfies the required conservation laws at the failure boundary, as well as in the surrounding linear zone. Comparing these results with previous representations of earthquake sources, in particular the "stress drop", "stress glut", "boundary stress pulse" or "dislocation" representations, leads us to conclude that none of these "models" is generally appropriate, although the latter two may be adequate approximations in certain circumstances since they may, for some failure modes, approximate the appropriate dislocation-stress pulse equivalent defined here.

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Appendix 1 - Magnitude of Seismic Radiation Effects Due to Induced Gravity Field Changes

In the initial state prior to failure, mechanical equilibrium requires:

$$\partial_k \tau_{lk}^{(I)} = \rho_I \, \partial_l \Phi^{(I)} \tag{A-1.}$$

where the gravitational potential Φ^I (x) and initial density ρ_I (x) satisfy

$$\nabla^2 \Phi^{(I)} = 4 \pi G \rho_I \tag{A-2a.}$$

or,

$$\Phi^{(I)} = 4 \pi G \int_V \frac{\rho_I}{|\mathbf{x} - \mathbf{x}'|} d^3x' \tag{A-2b.}$$

Here rotational forces for the planetary body are neglected as small relative to gravity.

The final state of the medium will be different after the earthquake but will also satisfy equilibrium conditions and (changed) boundary conditions, so that

$$\left.\begin{aligned} \partial_k \tau_{lk}^{(F)} &= \rho_F \, \partial_l \, \Phi^{(F)} \\ \nabla^2 \Phi_F &= 4 \pi G \rho_F \end{aligned}\right\} \tag{A-3.}$$

and therefore

$$\partial_k \left[\tau_{lk}^{(F)} - \tau_{lk}^{(I)} \right] = \rho_F \, \partial_l \, \Phi^{(F)} - \rho_I \, \partial_l \, \Phi^{(I)}$$

Noting that the left side of this equation involves the stress drop, then

$$\gamma_l^v \equiv \partial_k T_{lk} = \rho_F \, \partial_l \, \Phi^{(F)} - \rho_I \, \partial_l \, \Phi^{(I)} \tag{A-4.}$$

where γ_l^v is the spatial source term used by Gilbert to represent the seismic source "associated" with an earthquake.

Now setting

$$\left.\begin{aligned} \rho_F &= \rho_I + \delta\rho \\ \Phi_F &= \Phi_I + \delta\Phi \end{aligned}\right\} \tag{A-5.}$$

so that

$$\nabla^2 \Phi_F = \nabla^2 \Phi_I + \nabla^2 (\delta\Phi) = 4 \pi G (\rho_0 + \delta\rho)$$

then we have immediately:

$$\nabla^2 (\delta\Phi) = 4\pi G (\delta\rho) \tag{A-6a.}$$

Therefore the change in the gravitational potential is given by

$$\delta\Phi = 4\pi G \int_V \frac{\delta\rho}{|\mathbf{x} - \mathbf{x}'|} d^3x' \tag{A-6b.}$$

Further, by the equation of continuity

$$\delta\rho = -\nabla \cdot (\rho_I \mathbf{s}) \tag{A-7.}$$

where s denotes the displacement field produced by the earthquake.

Inserting (A-5.) in (A-4.), and using (A-6b.) and (A-7.) in the result, gives:

$$\gamma^v = -4\pi G \left\{ \rho_I \nabla \left[\int_V \frac{\nabla \cdot (\rho_I \mathbf{s})}{|\mathbf{x} - \mathbf{x}'|} d^3x' \right] \right.$$

(A-8.)

$$\left. + \nabla \cdot (\rho_I \mathbf{s}) \nabla \left[\int_V \frac{\rho_I}{|\mathbf{x} - \mathbf{x}'|} d^3x' \right] + \nabla \cdot (\rho_I \mathbf{s}) \nabla \left[\int_V \frac{\nabla \cdot (\rho_I \mathbf{s})}{|\mathbf{x} - \mathbf{x}'|} d^3x' \right] \right\}$$

Since we are interested in the order of magnitude of the source term γ^v, then the initial density can be approximated as a constant, ρ, in the integrals and in this case (A-8.) has the approximate value:

$$\gamma^v \simeq -4\pi G \rho^2 \left\{ (1 + \nabla \cdot \mathbf{s}) \nabla \left[\int_V \frac{\nabla \cdot \mathbf{s}}{|\mathbf{x} - \mathbf{x}'|} d^3x' \right] + (\nabla \cdot \mathbf{s}) \nabla \left[\int_V \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|} \right] \right\} \tag{A-9}$$

Noting that the divergence of the displacement field is the dilatation and that it is small compared to unity, that is observationally of maximum value near 10^{-3} for earthquakes at near distances, then the factor $(1 + \nabla \cdot \mathbf{s})$ in the first term in (A-9.) can be approximated by unity. Therefore:

$$\gamma^v \simeq -4\pi G \rho^2 \left\{ \left[\int_V \frac{\nabla \cdot \mathbf{s}}{|\mathbf{x} - \mathbf{x}'|} d^3x' \right] + (\nabla \cdot \mathbf{s}) \nabla \left[\int_V \frac{d^3x'}{|\mathbf{x} - \mathbf{x}'|} \right] \right\} \tag{A-10.}$$

To this point the results are general, in that the volume of integration for the gravitational effect can be taken as the entire volume exterior to the failure zone (the volume denoted V_1 in the text) or the volume interior to the failure zone boundary denoted V' in the text, or both. Gilbert's representation involves an integration over V_1, while that of Backus and Mulcahy involves an integration over V'.

The representation of the force equivalent, **f**, used by Gilbert is given by:

$$\mathbf{f} \equiv \gamma^{v} H(t) \ ; \ \mathbf{r}' \varepsilon V_1$$

where H(t) is a step function. The designation of the region of applicability, $\mathbf{r}' \varepsilon V_1$, means that this equivalent is defined over the volume exterior to the failure zone. On the other hand, the "stress glut" formation of Backus and Mulcahy has this same form in the low frequency limit, but with the region of applicability for the equivalent defined to be over V', the failure zone.

Considering the case appropriate to Gilberts formulation, for which (A-10.) is directly applicable, and noting that

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \begin{cases} \sum\limits_{l,m} \left[\dfrac{4\pi}{2l+1}\right] \left[\dfrac{r'^{l}}{r^{l+1}}\right] Y^{*}_{lm}(\theta,\phi)\, Y_{lm}(\theta',\phi') \ ; r > r' \\ \sum\limits_{l,m} \left[\dfrac{4\pi}{2l+1}\right] \left[\dfrac{r^{l}}{r'^{l+1}}\right] Y^{*}_{lm}(\theta,\phi)\, Y_{lm}(\theta',\phi') \ ; r < r' \end{cases}$$

where $Y_{lm}(\theta,\phi) = P_l^m(\cos\theta)\, e^{im\phi}$, with P_l^m the associated Legendre function and where Y^{*}_{lm}, is the complex conjugate of Y_{lm}, then in (A-10.) we have:

$$\nabla\left[\int\limits_{V_1} \frac{d^3x'}{|\mathbf{x}-\mathbf{x}'|}\right] = \nabla\left[\frac{4\pi}{r}\int_0^r r'^2\, dr' + 4\pi\int_r^R r'\, dr'\right] = -4\pi/3\, r\, \hat{e}_r$$

where $\hat{e}_r$ is the unit vector in the radial direction. Here the integration volume V_1 has been taken to extend from the origin to a large distance R from the failure zone, as required by the finite dimensions of the planet. Similarly, the integration has been taken through the failure zone in order to give an upper estimate for the size of this term.

Likewise the magnitude of the other integrals term in (A-10.) can be estimated by noting that the dilatation $\Theta = \nabla \cdot \mathbf{s}$ is a harmonic function (*eg*. Love, 1944). For the region outside the failure zone this harmonic function is always of the form

$$\Theta = \sum_n \left[\frac{1}{r^{n+1}}\right] S_n(\theta\,,\phi) \qquad \text{(A-11.)}$$

where

$$S_n(\theta\,,\phi) = \sum_{m=o}^{n} \left[\alpha_{nm} \cos m\,\phi + \beta_{nm} \sin m\,\phi\right] P_n^m(\cos\theta)$$

with the constants α_{nm} , β_{nm} dependent on the failure zone geometry and the initial stress state of the medium. (See Archambeau, 1964 and 1968 for examples.) Thus, with $|\mathbf{x} - \mathbf{x}'| \equiv r^*$, the second integral term in (A-10.) can be expressed as:

$$\int_{V_1} \frac{\nabla \cdot s}{|\mathbf{x} - \mathbf{x}'|}\, d^3x' = \sum_n \left\{ \int_{V_1} \frac{1}{r^*} \left[\frac{S_n(\theta'\,,\phi')}{r'^{n+1}}\right] r'^2 \sin\theta'\, d\theta'\, d\phi'\, dr' \right\}$$

Now, employing a method due to Love (1944), and applied in similar circumstances by Archambeau (1968, p. 255) the integration over V_1 can be rearranged to be performed over surfaces of constant r^* around the "observation point" at $\mathbf{r}$ and then (finally) over r^*, to cover all of V_1. We therefore obtain, after extending the region of integration to include the failure zone region as well as the surrounding medium:

$$\int_{V_1} \frac{\nabla \cdot s}{|\mathbf{x} - \mathbf{x}'|}\, d^3x' = \sum_n \int_0^R \frac{dr^*}{r^*} \int_{\Omega^*} \frac{S_n(\theta'\,,\phi')}{(r')^{n+1}}\, da'$$

where $R \geq r$ and Ω^* is the spherical surface of radius $r^* = |\mathbf{r} - \mathbf{r}'|$. Since

$$\int_{\Omega^*} \frac{S_n(\theta'\,,\phi')}{(r')^{n+1}}\, da' = \begin{cases} 4\pi\, r^{*2} \left[\dfrac{S_n(\theta\,,\phi)}{r^{n+1}}\right] ; \; r^* < r \\ 0 \,;\; r^* > r \end{cases}$$

by an extension of the mean value theorem for harmonic functions (see Archambeau, 1964 for details), then

$$\int_{V_1} \frac{\nabla \cdot \mathbf{s}}{|\mathbf{x}-\mathbf{x}'|}\, d^3x' = 4\pi \sum_n \left[\int_0^r r^{*}\, dr^{*} \right] \frac{S_n(\theta , \phi)}{r^{n+1}} = 2\pi\, r^2\, \Theta$$

Consequently the second term in (A-10.) becomes:

$$\nabla \left[\int_{V_1} \frac{\nabla \cdot \mathbf{s}}{|\mathbf{x}-\mathbf{x}'|}\, d^3x' \right] = 2\pi\, \nabla(r^2\, \Theta) = 2\pi\, r^2\, \nabla\Theta + 4\,\pi\, r\, \Theta\, \hat{e}_r$$

Now, collecting results from the integral evaluations one has for the equivalent force in (A-10.):

$$\gamma^V = -\, 8\pi\, G\, \rho^2 \left[4/3\, r\, \Theta\, \hat{e}_r + r^2\, \nabla\Theta \right] \tag{A-12.}$$

To directly estimate the contribution of such a source term to the wave field we observe that the dynamic displacement field, **u**, is given by:

$$u_k(\mathbf{x} , t) = \frac{1}{4\pi} \int_0^{\infty} dt_0 \int_{V_1} f_l\, (\mathbf{x}_o , t_o)\, G_l^k\, (\mathbf{x} , \mathbf{x}_0\, ; t_o - t)\, d^3x_0$$

with $f_l = \gamma_l^V\, (x_0)\, H(t_o)$ and G_l^k a tensor Greens function for the medium. Alternately, the dilatation and rotation of the dynamic field is given, in the spectral domain, by the simpler relations (*eg*. Archambeau, 1968):

$$\left.\begin{aligned} \bar{\chi}_0\, (\mathbf{x} , \omega) &= \frac{1}{4\pi\, \rho v_p^2} \int_0^{\infty} H\, (t_o)\, e^{-i\omega t_o}\, dt_o \int_{V_1} (\nabla \cdot \gamma^V)\, \frac{e^{-ik_p r^{*}}}{r^{*}}\, d^3x_0 \\ \bar{\boldsymbol{\chi}}\, (\mathbf{x} , \omega) &= \frac{1}{4\pi\, \rho v_s^2} \int_0^{\infty} H\, (t_o) e^{-i\omega t_o}\, dt_o \int_{V_1} \frac{1}{2}\, (\nabla \times \gamma^V)\, \frac{e^{-ik_s r^{*}}}{r^{*}}\, d^3x_o \end{aligned}\right\} \tag{A-13.}$$

with infinite space scalar Greens functions used, where $r^{*} \equiv |\, \mathbf{r} - \mathbf{r}_0\, |$ and $k_p = \omega / v_p$, $k_s = \omega / v_s$. Here $\bar{\chi}_0$ denotes the dilatation while $\bar{\boldsymbol{\chi}} = (\bar{\chi}_1 , \bar{\chi}_2 , \bar{\chi}_3)$ denotes the rotation field with Cartesian components χ_j. These physical potentials should be of the order of the strain changes observed from an earthquake if this source term is the proper equivalent for an earthquake source. These potentials are therefore convenient for estimation purposes.

From (A-12.) we have:

$$\nabla \cdot \gamma^{v} = -32\,\pi^2 \rho^2 G \left[r\,\Theta + \frac{5}{6}\, r\, \frac{\partial \Theta}{\partial r} + \frac{1}{3}\,\Theta \right]$$

$$\nabla \times \gamma^{v} = \frac{16}{3}\,\pi^2 \rho^2 G \left[r\,\nabla\,\Theta \times \hat{e}_r \right] = \frac{16\pi^2}{3}\,\rho^2 G \left[\frac{\partial \Theta}{\partial \theta}\,\hat{e}_\phi - \frac{1}{\sin\theta}\,\frac{\partial \Theta}{\partial \phi}\,\hat{e}_\theta \right]$$

where Θ has the form given in (A-11.). Inserting these relations along with the expansion for Θ into (A-13.) and introducing the spherical wave the expansions for the scalar Greens functions, of the form

$$\frac{e^{-ikr^{*}}}{r^{*}} = -\,ik \sum_{l=0}^{\infty} (2\,l+1)\,P_l\,(\cos\gamma)\,j_l\,(k\,r')\,h_l^{(2)}\,(k\,r) \;\; ; r > r'$$

gives:

$$\tilde{\chi}_o^{(G)}\,(\mathbf{r}\,,\omega) = \frac{32\pi^2\,\rho G}{v_p^3} \sum_n S_n(\theta\,,\phi)\,h_n^{(2)}\,(k_p\,r) \int_0^R \left[\left[\frac{1}{r'}\right]^{n-2} - (\frac{5}{6}n + \frac{1}{2}) \left[\frac{1}{r'}\right]^{n-1} \right] j_n(k_p r')\,dr'$$

$$\tilde{\chi}^{(G)}\,(\mathbf{r}\,,\omega) = -\,\frac{8\pi^2\,\rho G}{3v_s^3} \sum_n \mathbf{S}_n(\theta\,,\phi)\,h_n^{(2)}\,(k_s\,r) \int_0^R \left[\frac{1}{r'}\right]^{n-1} j_n(k_s\,r')\,dr'$$

Here $\mathbf{S}_n(\theta\,,\phi)$ denotes a vector with components of the same type as $S_n(\theta\,,\phi)$. In deriving these expressions we have evaluated the transform of the step function H(t) and have used the orthogonality relation

$$\int_0^{2\pi}\!\!\int_0^{\pi} P_l\,(\cos\gamma)\,S_n\,(\theta'\,,\phi')\,\sin\theta'\,d\theta'\,d\phi' = \frac{4\pi}{2n+1}\,S_n\,(\theta\,,\phi)\,\delta_{ln}\,,$$

with γ the angle betweend the vectors **r** and **r′**. Again the range of integraton has been extended through the failure zone to the origin, in order to obtain maximum estimates.

In evaluating the radial integrals in the expressions for the wave field induced by gravity changes we can further simplify results and obtain an accurate estimate by taking the upper limit (R) to be very large, or infinite, and using the integral relation:

$$\int_0^{\infty} x^{\mu-1}\,J_\nu\,(a\,x)\,dx = 2^{\mu-1}\,a^{-\mu}\,\frac{\Gamma(1/2\,\nu + 1/2\,\mu)}{\Gamma(1 + \nu/2 - \mu/2)}$$

valid for $-\operatorname{Re} \upsilon < \operatorname{Re} \mu < 3/2$. We have in this case; for $n \geq 2$:

$$\tilde{\chi}_0^{(G)}(\mathbf{r}, \omega) \leq \frac{32\pi^2 \rho G}{v_p^3} \sum_n \left[\frac{\pi k_p^{n-3}}{2^n \Gamma(n)} - \frac{\sqrt{\pi}(5/6\, n + 1/2) k_p^{n-2}}{2^n \Gamma(n + 1/2)} \right] S_n(\theta, \phi)\, h_n^{(2)}(k_p r) \qquad \text{(A-14a.)}$$

$$\tilde{\chi}^{(\ i)} \leq -\frac{8\pi^2 \rho G}{3v_s^3} \sum_n \left[\frac{\sqrt{\pi}\, k_s^{n-2}}{2^n \Gamma(n + 1/2)} \right] S_n(\theta, \phi)\, h_n^{(2)}(k_s r) \qquad \text{(A-14b.)}$$

The magnitude of the dilatation is larger than, but of the same order as, the components of the rotation vector field. Therefore it is sufficient to consider the size of only $\tilde{\chi}_0^{(G)}$, in (A-14a.), relative to observed dynamic strain changhes accompanying earthquakes. To obtain a numerical estimate of $\tilde{\chi}_0^{(G)}$, it is sufficient to consider the (extreme) case in which a fluidized spherical failure volume is produced in an initially shear stressed medium. In this case we have (Archambeau, 1968; Landau and Lifshitz, 1959) the specific form for Θ in (A-11.):

$$\Theta(\mathbf{r}) = \frac{1}{r^3} S_2(\theta, \phi)$$

with

$$S_2(\theta, \phi) = \sum_{m=0}^{2} \left[\alpha_{2m} \cos m\phi + \beta_{2m} \sin m\phi \right] P_2^m(\cos\theta)$$

where

$$(\alpha_{2m}) = \frac{5(1 - 2\sigma)}{\mu(7 - 5\sigma)} R_0^3 \left(0 \ \ \tau_{13}^{(I)} \ \ 0 \right) ; m = 0, 1, 2$$

$$(\beta_{2m}) = \frac{5(1 - 2\sigma)}{\mu(7 - 5\sigma)} R_0^3 \left(0 \ \ \tau_{23}^{(I)} \ \ \tau_{12}^{(I)}/2 \right) ; \ m = 0, 1, 2$$

Here $\tau_{ij}^{(I)}$ denotes an initial homogeneous shear prestress, R_0 is the failure zone radius, σ is the Poissons ratio in the medium and μ the rigidity. Thus, for this simple case for which we can expect to obtain the largest gravity induced radiation effect associated with failure in a stressed medium;

$$\tilde{\chi}_0^{(G)}(\mathbf{r}, \omega) \leq 32 \quad G \left[\frac{5(1 - 2\sigma)}{7 - 5\sigma} \right] \left(\frac{R_0}{v_p^3} \right) \left[\frac{\pi}{2k_p} - \frac{13}{9} \right] \left(\frac{\tau_{13}^{(I)}}{2\mu} \right) \cdot h_2^{(2)}(k_p r)\, P_2^1(\cos\theta) \cos\phi$$

At a frequency of 1Hz and at a distance of about one wave length at 1Hz (*ie.* about 5 km.) from the source origin, we have:

$$\left|\tilde{\chi}_0^{(G)}\right| \leq 32\,\pi^2\,\rho\,G\left[\frac{5(1-2\sigma)}{7-5\sigma}\right]\left[\frac{R_0}{v_p}\right]\left|\frac{\tau_{13}^{(1)}}{2\mu}\right|\left[\frac{1}{20}\right] \qquad \text{(A-16.)}$$

Taking a large failure zone such that $R_0 / v_p \simeq 1$, so that a distance of one wave length at one hertz is close to the failure zone boundary, and an initial shear strain such that $|\tau_{13}^{(1)}/2\mu| \simeq 10^{-3}$, which is typical of the level producing failure (*ie.* hundred of bars of shear stress or less), then with $\sigma \simeq 1/4$ and $\rho \sim 5$ gm/cm^3 as representative values, we have

$$|\tilde{\chi}_0^{(G)}| < 10^{-8}$$

where this value of $|\tilde{\chi}_0^{G}|$ has the units and the magnitude of the dynamic strain field, in this case near the failure zone boundary. Similarly, the spectral density of the rotation components in (A-14b.) will be such that

$$|\tilde{\chi}_j^{(G)}| < 10^{-8}$$

at 1 Hz and at a distance of one wavelength.

The order of magnitude bounds for these quantities, all of which should be of the order of the spectral levels of the dynamic strain changes in the medium if they are to produce the seismic radiation from a tectonic source, are clearly many orders of magnitude less than those observed. In particular, we can expect them to be of about the order of $(\omega^{-1})\ |\tau_{13}^{(1)}/2\mu|$ near the failure zone, which in the present case implies a level of about 10^{-4} at 1 Hz. The values predicted for gravity change induced effects are therefore at least four orders of magnitude too small, as would be expected. The same analysis, when applied to the effects of gravity changes *inside* the failure zone alone such as are implicit in the "stress glut" formulation, produce similar, but even smaller, effects.

The origin of the small gravity effect is associated with the small value of the "seismo-gravity coupling" factor, $K_G \doteq 8\pi^2\rho^2 G$, appearing in the basic equivalent force field relation of (A-12.). Here,

$G = 6.67\times10^{-8}$ (cgs units) and so $K_G \sim 10^{-4}$. The coupling factor therefore scales (down) the dynamic field by about 10^{-4} and produces the small gravity related effects just described.

By way of contrast and in order to show the role of the coupling more precisely, the magnitude of the seismic radiation due to stress relaxation around such a spherical failure zone is, to the same approximation as was used for the gravity effect calculation (see Archambeau, 1972 for exact results):

$$\tilde{\chi}_0^{(R)}(\mathbf{r}, \omega) \simeq \left[\frac{5(1-2\sigma)}{7-5\sigma}\right]\left(\frac{R_0}{v_p}\right)^3\left(\frac{\tau_{13}^{(I)}}{2\mu}\right)\left(\frac{\omega^2}{6}\right) h_2^{(2)}(k_p r)\, P_2^1(\cos\theta)\cos\phi$$

with similar relations for the rotation field components. At 1 Hz and at one wave length from the source origin, we have for the magnitude of the dilatation field:

$$\left|\tilde{\chi}_0^{(R)}(\mathbf{r}, \omega)\right| \simeq \left[\frac{5(1-2\sigma)}{7-5\sigma}\right]\left(\frac{R_0}{v_p}\right)^3\left|\frac{\tau_{13}^{(I)}}{2\mu}\right|$$

When R_0/v_p is near unity then this magnitude estimate , at a distance near the failure boundary and for $\sigma \simeq 1/4$, is of the order of the prestrain given by $|\tau_{13}^{(I)}/2\mu|$. Thus, the dilatation at 1 Hz (and likewise the rotation field) is of the order of the prestrain, or of the order 10^{-3}, if we use the same prestrain level assumed in the gravity effect calculation. We also observe that this radiation field has the same spatial dependence and scales with source dimensions, prestrain and medium properties in the same way as the gravity induced effect, although its frequency dependence is somewhat different. This is as would be expected since it is this direct radiation effect, associated with the relaxation of the prestress field, that produces the related gravity effect. However, the ratio of the magnitudes of the two effects is seen to be such that:

$$|\tilde{\chi}_0^{(G)}| / |\tilde{\chi}_0^{(R)}| < \frac{4K_G}{\rho} \simeq 10^{-4}$$

at 1 Hz and at all spatial locations in the medium. A similar relationship holds at other frequencies as well, as can be verified. However a demonstration at 1 Hz is sufficient. Thus, the gravity effects are clearly negligible compared to direct stress relaxation produced radiation.

References

Archambeau, C.B. (1964), *Elastodynamic Source Theory*, Ph.D. Thesis, Calif. Inst. of Tech., Pasadena. Calif.

Archambeau, C. B. (1968), General theory of elastodynamic source fields, *Rev. Geophys., 16*, pp. 241-288.

Archambeau, C.B. (1972), The theory of stress wave radiation from explosions in prestressed media, *Geophys. J.R. Astr. Soc., 29*, pp. 329-366.

Archambeau, C. B. and J. B. Minster (1978), Dynamics in prestressed media with moving phase boundaries: A continuum theory of failure in solids, *Geophys. J. R. Astr. Soc., 52*, pp. 65-96.

Archambeau, C.B. and G. A. Dilts (1989), Multipole moment expansions for dynamical representations of seismic sources, In preparation, to be submitted for publication, Geophys. J.R. Astr. Soc.

Backus, G. and M. Mulcahy (1976a), Moment tensors and other phenomenological descriptions of seismic sources, I. Continuous displacements, *Geophys. J. R. Astr. Soc., 46*, pp. 341-461.

Backus, G. and M. Mulcahy (1976b), Moment tensors and other phenomenological descriptions of seismic source - II. Discontinuous displacements, *Geophys. J. R. Astr. Soc., 45*, pp. 301-329.

Ben Menahem, A. and S. Singh (1972), Computations of models of elastic dislocations in the earth: in *Methods of Compuational Physics, 12*, Academic Press, New York, pp. 300-372.

Ben Menahem, A. and S. J. Singh (1981), *Seismic Waves and Sources*, Springer-Verlag, New York.

Dilts, G. A. (1985), *The dynamical equilibrium: A mathematical model for the Dynamical description of the permanent deformation of an elastic material caused by micro-fracture or plastic flow*, Ph.D. Thesis Physics, Dept., U. of Colorado, Boulder, Colo.

Eringen, A. C. (1985), *Continuum Physics* (A.C. Eringen, editor), Vol.II, Academic Press, N.Y.

Eshelby, J. D. (1957), The determination of the elastic field of an ellipsoidal inclusion and related problems, *Proc. Roy. Soc. Lond. A, 241*, pp. 376-396.

Geller, R. J. (1976), Body force equivalents for stress-drop seismic sources, *Bull. Seism. Soc. Amer., 66*, pp. 1801-1804.

Gilbert, F. (1973), Derivation of source parameters from low frequency spectra, *Phil. Trans. R. Soc. Lond. A., 274*, pp. 369-371.

Gilbert, F. (1970), Excitation of the normal modes of the earth by earthquake sources, *Geophys. J. R. Astr. Soc., 22*, pp. 223-226.

Gilbert, F. and A. M. Dziewonski (1975), An application of normal mode theory to the retrieval of structure parameters and source mechanisms from seismic spectra, *Phil. Trans. R. Soc. Lond., 278*, pp. 187-269.

Harvey, D. (1981), Seismogram synthesis using normal mode superposition: The locked mode approximation, *Journal of the Royal Astron. Soc., 66*, pp. 37-61.

Landau, L. D. and E. M. Lifshitz (1951), *Theory of elasticity*, Addison-Wesley Publishing Co.

Love, A. E. H. (1944), *A treastise on the mathematical theory of elasticity*, Dover.

Luh. P. C. and A. M. Dziewonski (1976), Theoretical normal mode spectra of a rotating elliptical earth, *Geophys. J. R. Astr. Soc., 45*, pp. 617-645.

McCowan, D. W. (1976), Moment tensor representation of surface wave sources, *Geophys. J. R. Astr. Soc., 44*, pp. 595-599.

McCowan, D. W. and A. M. Dziewonski (1977), An application of the energy-moment tensor relation to estimation of seismic energy radiated by point and line sources, *Geophys. J. R. Astr. Soc., 51*, pp.

531-544.

Minster, J.B. (1974), *Elastodynamics of failure in a contimunm*, Ph.D. Thesis, Calif. Inst. of Tech., Pasadena, Calif.

Stevens, J.L. (1980), *"Seismic Stress Relaxation Phenomena in an Inhomogeneously Prestressed Medium"*, Ph.D. Thesis, Physics Dept., U. of Colorado.

Stevens, J.L. (1980), "Seismic radiation from the sudden creation of a spherical cavity in an arbitrarily prestressed elastic medium, *Geophys. J. R. Astr. Soc.*, *61*, 303-328.

Stump, B.W. and L.R. Johnson (1982), Higher-degree moment tensors-The importance of source finiteness and rupture propagation on seismograms, *Geophys. J.R. Astr. Soc.*, *69*, 721-743.

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